Note Title 9/27/2005

corrected

09/26/05

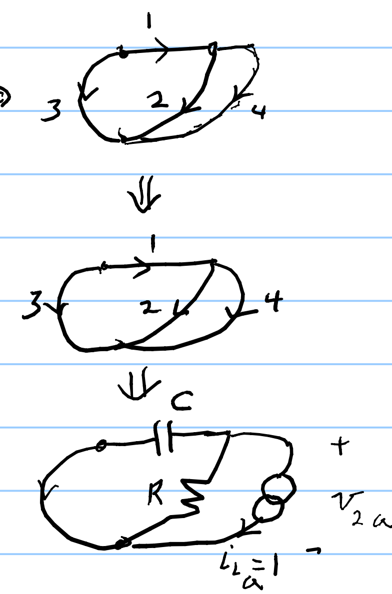
$$-\Delta v_L = -\Delta\left(\frac{v_L}{v_s}\right) = -v_{int}^T \, \Delta Y^T \, v_{a\,int}$$

$$= -\Delta\left(\text{voltage transfer function}\right)$$

Example:

$v_{in} = e_1$

$\Rightarrow$



Find $\frac{\partial\, v_L/v_{in}}{\partial G}$

$G = 1/R$

$$Y = \begin{bmatrix} Cs & 0 \\ 0 & G \end{bmatrix}$$

$$\Delta Y = \begin{bmatrix} 0 & 0 \\ 0 & \Delta G \end{bmatrix}$$

$$v_{in}^T \Delta Y^T v_{int_a} = [v_1 \; v_2] \begin{bmatrix} 0 & 0 \\ 0 & \Delta G \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_a = -\Delta\left({v_L}/{v_{in}} \right)$$

$$\frac{\Delta\left({v_L}/{v_{in}} \right)}{\Delta G} = -v_2 v_{2a}$$

here $v_2 = v_L = \dfrac{R_2}{R_2 + \frac{1}{sC}} \cdot e_1 = \dfrac{R_2 sC}{R_2 sC + 1}$

next (choosing $i_{L_a} = 1$); $y_{i_{L_a}} = \dfrac{1}{G + sC} = \dfrac{R}{1 + sCR}$, $v_{2a} = -y_{i_{L_a}} i_{L_a}$

$$v_{2a} = \frac{-R}{1 + sCR} \; ; \quad \frac{\partial\, {v_L}/{v_{in}}}{\partial G} = + v_2 v_{2a} = +\left(\frac{RsC}{RsC + 1} \right)\left(\frac{-R}{1 + sCR} \right)$$

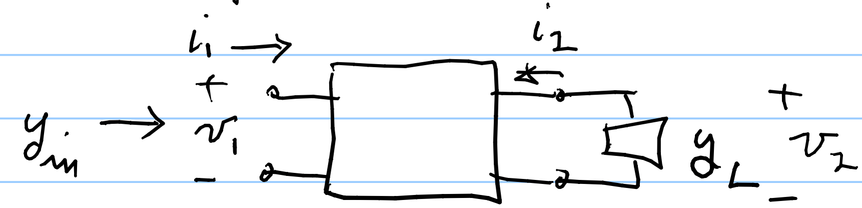
$$= \frac{-sC}{(sC + G)^2} = \frac{-R^2 sC}{(1 + RCs)^2}$$

did not take any derivatives – just analyze the circuit

$$\frac{v_L}{v_{in}} = \frac{R}{R + \frac{1}{sC}} = \frac{sC}{G + sC}$$

$$\frac{\partial\left(\frac{v_L}{v_{in}}\right)}{\partial G} = \frac{AC(-1)}{(G+AC)^2} = -\frac{AC}{(G+AC)^2}$$

Loaded 2-ports – assume an admittance matrix for the 2-port



$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = Y \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$y_{in} = \left.\frac{i_1}{v_1}\right|_{\text{when load}} \qquad \therefore \quad i_2 = -y_L v_2$$

$$\begin{bmatrix} i_1 \\ -y_L v_2 \end{bmatrix} = \begin{bmatrix} y_{11} v_1 + y_{12} v_2 \\ y_{21} v_1 + y_{22} v_2 \end{bmatrix}$$

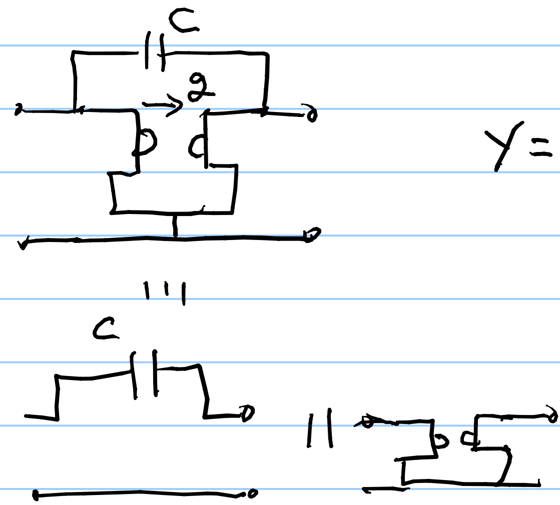
last eq. $-y_L v_2 - y_{22} v_2 = y_{21} v_1 \Rightarrow v_2 = -(y_L + y_{22})^{-1} y_{21} v_1$

$$i_1 = \left[y_{11} - y_{12}(y_L + y_{22})^{-1} y_{21} \right] v_1 \qquad Y_{in} = y_{11} - y_{12}(y_L + y_{22})^{-1} y_{21}$$

if a 1-port load $\Rightarrow$ $y_{in} = Y_{in} = (y_{11}[y_L + y_{22}] - y_{12}y_{21})/(y_L + y_{22})$

$$= \frac{(y_{11} y_L + \Delta y)}{y_{22} + y_L}$$

Ex:



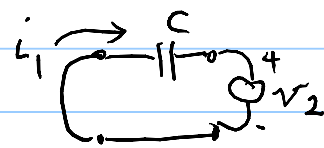
$$Y = Y_{cap} + Y_{gy} = \begin{bmatrix} sC & -sC \\ -sC & sC \end{bmatrix} + \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$$

$$= \begin{bmatrix} sC & -sC+g \\ -sC-g & sC \end{bmatrix}$$

$$y_{in} = \frac{sC y_L + [(sC)^2 + (sC+g)(-sC+g)]}{sC + y_L}$$

$$= \frac{sC y_L + g^2}{sC + y_L}$$

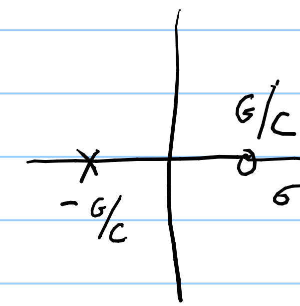
for $y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1 = 0}$



but to find y_L as a function of y_{in}: $(sC + y_L)\, y_{in} = sC\, y_L + g^2$

$$sC\, y_{in} \mp g^2 = (sC - y_{in})\, y_L \Rightarrow y_L = \frac{sC\, y_{in} \mp g^2}{sC - y_{in}}$$

Ex: $y_L = G$; if $y_{in} = \dfrac{sCG - g^2}{sC + G}$ if $g = G$ then $\dfrac{y_{in}}{G} = \dfrac{sC - G}{sC + G}$

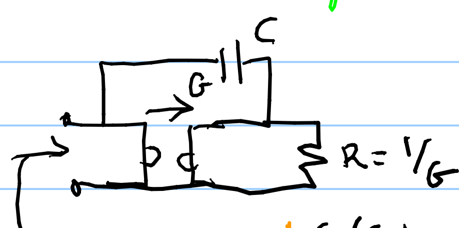


for $s = j\omega$

$$\left|\frac{y_{in}(j\omega)}{G}\right| = \left|\frac{j\omega C - G}{j\omega C + G}\right| = \frac{\sqrt{(\omega C)^2 + G^2}}{\sqrt{(\omega C)^2 + G^2}} = 1 \text{ for all } \omega$$

∴ called all-pass

for



$$y_{in} = G\left(\frac{s + G/C}{s + G/C}\right) = G$$ or clearly all-pass

Look at $Ev\, y_{in}(s)$; $a(s)$ has the even part $\frac{a(s)+a(-s)}{2} = Ev(a(s))$

" " odd part $\frac{a(s)-a(-s)}{2} = Od(a(s))$

$a(s) = Ev(a(s)) + Od(a(s))$

Ex: $Ev\, y_{in}(s) = \frac{y_{in}(s)+y_{in}(-s)}{2} = \frac{\frac{sCG-g^2}{sC+G} + \frac{-sCG-g^2}{-sC+G}}{2}$ for $y_{in} = \frac{sCG-g^2}{sC+G}$.

$2\, Ev\, y_{in} = \frac{(sCG)(-sC) + sCG^2 - g^2(-sC) - g^2 G}{(sC+G)(-sC+G)} - \frac{(sCG)(sC) - sCG^2 - sCg^2 - Gg^2}{(sC+G)(-sC+G)}$

$= \frac{2\left(-(sC)^2 G - Gg^2\right)}{(sC+G)(-sC+G)} \Rightarrow$

$Ev\, y_{in} = \frac{-((sC)^2+g^2)G}{-(sC)^2+G^2} = G\frac{((sC)^2+g^2)}{(sC)^2-G^2}$ zeros of $Ev\, y_{in} \Rightarrow sC = \pm j g$