09/21/05

$A = \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix}$ p. 244+

$$\lambda 1_2 x = \lambda x = Ax \; ; \; (\lambda 1_2 - A)x = 0$$

solution for $\lambda$ = eigenvalue if and only if

$\det(\lambda 1_2 - A) = 0$ $\quad \det\begin{bmatrix} \lambda+2 & -3 \\ 0 & \lambda-4 \end{bmatrix} = (\lambda+2)(\lambda-4)$

$= \lambda^2 - 2\lambda - 8\lambda^0$

$\lambda_1 = -2, \; \lambda_2 = 4$ are eigenvalues

note if $\lambda \to A$ $\quad Ax = Ax$

$\therefore$ if $\lambda \to A$ in $\lambda^2 - 2\lambda - 8\lambda^0 \Rightarrow A^2 - 2A - 8 1_2 = 0_2$

$A^2 = \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 0 & 16 \end{bmatrix}$

$-2A \qquad = \begin{bmatrix} +4 & -6 \\ 0 & -8 \end{bmatrix}$

$-8A^0 \qquad = \begin{bmatrix} -8 & 0 \\ 0 & -8 \end{bmatrix}$

add these $= \begin{bmatrix} 4+4-8 & 6-6+0 \\ 0+0+0 & 16-8-8 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

a matrix satisfies its characteristic equation

$\det(\lambda 1_n - A) = 0$ $\quad$ if $A$ is $n \times n$

$= P(\lambda) = \lambda^n + \cdots$ $\quad$ a polynomial of degree $n$

useful as "any" function of $A$ will be a polynomial of degree $n$

Ex: $f(A) = A^3 = (A^2)\cdot A = (2A + 8 1_2)A = 2A^2 + 8A = 2(2A + 8 1_2) + 8A$

$= 12A + 16 1_2$

Eq 6.2-3: $e^A \Rightarrow e^{\lambda_1} = \alpha_0 + \alpha_1 \lambda_1 = e^{-2}$ ; $e^A = \alpha_0 1_2 + \alpha_1 A$

$e^{\lambda_2} = \alpha_0 + \alpha_1 \lambda_2 = e^4$

$$\begin{bmatrix} 1 & \lambda_1 \\ 1 & \lambda_2 \end{bmatrix}\begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix} = \begin{bmatrix} e^{-2} \\ e^4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix} ; \begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} e^{-2} \\ e^4 \end{bmatrix}$$

$$e^A = e^{\begin{bmatrix}-2&3\\0&4\end{bmatrix}} = \frac{4e^{-2}+2e^{4}}{6}1_2 + \frac{-e^{-2}+e^{4}}{6}\begin{bmatrix}-2&3\\0&4\end{bmatrix}$$

$$e^{At} = e^{\begin{bmatrix}-2&3\\0&4\end{bmatrix}t} = \frac{4e^{-2t}+2e^{4t}}{6}1_2 + \frac{-e^{-2t}+e^{4t}}{6}\begin{bmatrix}-2&3\\0&4\end{bmatrix} = \begin{bmatrix}1e^{-t} & -\frac{3}{6}e^{-2t}+\frac{3}{6}e^{4t}\\ 0 & 1.e^{4t}\end{bmatrix}$$

$$\dot{x} = Ax;\quad \mathcal{L}[\dot{x}] = sX - x_0 = AX;\quad X = (s1_2 - A)^{-1}x_0$$

$$X(s) = \begin{bmatrix}s+2 & -3\\ 0 & s-4\end{bmatrix}^{-1}x_0 = \frac{1}{(s+2)(s-4)}\begin{bmatrix}s-4 & 3\\ 0 & s+2\end{bmatrix}x_0$$

$$Z(s) = \frac{1}{(s+2)(s-4)}\begin{bmatrix}s-4 & 3\\ 0 & s+2\end{bmatrix} = \begin{bmatrix}\frac{1}{s+2} & \frac{3}{(s+2)(s-4)}\\ & \frac{1}{s-4}\end{bmatrix}$$

$$= \frac{1}{s+2}\begin{bmatrix}1 & 3/(-6)\\ 0 & 0\end{bmatrix} + \frac{1}{s-4}\begin{bmatrix}0 & a = 3/6\\ 0 & 1\end{bmatrix}$$

$$a = \left.\frac{3\,(s-4)}{(s+2)(s-4)}\right|_{s=4}$$

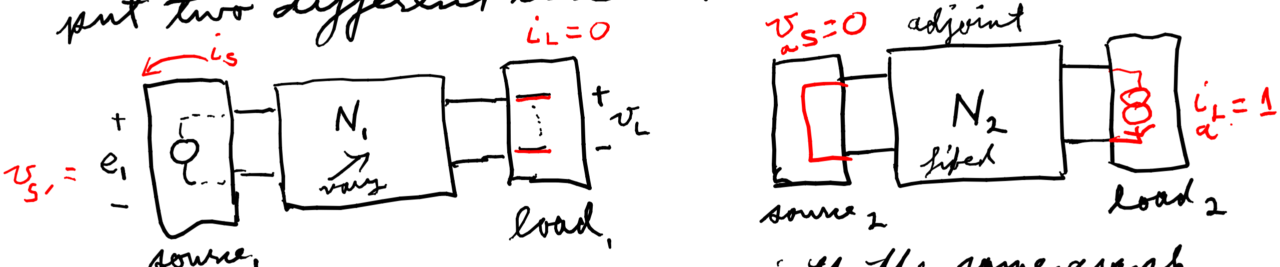
$$\dot{x} = Ax \Rightarrow e^{At}x_0 = x(t);\quad \mathcal{L}^{-1}[Z(s)] = e^{At}$$

$$e^{At} = e^{-2t}\begin{bmatrix}1 & -1/2\\ 0 & 0\end{bmatrix} + e^{4t}\begin{bmatrix}0 & 1/2\\ 0 & 1\end{bmatrix}$$

---- adjoint network (circuit) and use in finding derivatives (for sensitivities)

$i_b^T v_b = 0$ $\qquad i_{b_1}^T v_{b_2} - i_{b_2}^T v_{b_1} = 0$

put two different circuits on the same graph




both pictures with the same graph

assume 1st a circuit, & circuit 2 have admittance

$i_{b_1\,int}^T v_{b_2\,int} = i_{b_2\,int}^T v_{b_1\,int}$ $\qquad i_1 = Y_1 v_1$ ; $i_2 = Y_2 v_2$

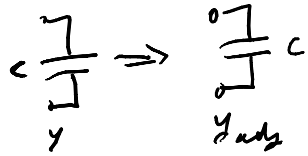
$\qquad\qquad\qquad\qquad\qquad\qquad Y_1 = Y \qquad Y_2 = Y_{adjoint}$

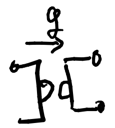
scalar $a = a^T$

$$v_{b_1}^T Y^T v_{b_2} = v_{b_2}^T Y_{adj} v_{b_1}$$

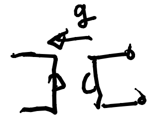
$$v_{b_2}^T Y v_{b_1} = v_{b_2}^T Y_{adj}^T v_{b_1}$$ for all $v_{b_1}$ & $v_{b_2}$ $\Rightarrow$ $Y = Y_{adj}^T$

or $Y_{adj} = Y^T$

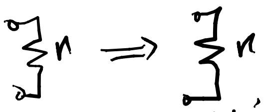




$Y = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$ $\Rightarrow$



$Y^T = Y_{adj} = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix}$



adjoint of a resistor

$$i_{b_1}^T v_{b_2} - i_{b_2}^T v_{b_1} = 0$$

$$i_S v_{S_a} + i_L v_{L_a} - i_{S_a} v_S - i_{L_a} v_L + \underbrace{i_{int}^T v_{int_a} - i_{int_a}^T v_{int}}_{v_{int}^T Y^T v_{int_a} - v_{int_a}^T Y_{adj}^T v_{int}} = 0$$

make a change in something in the original circuit, $\Delta$
say a component in the original circuit

$$\Delta\left(i_S v_{S_a} + i_L v_{L_a} - i_{aS} v_S - i_{aL} v_L\right) = \Delta v_{int}^T Y^T v_{a\,int} + v_{int}^T \Delta Y^T v_{a\,int} - \underbrace{v_{a\,int}^T Y_{adj}^T \Delta v_{int}}_{-\Delta v_{int}^T Y_{adj} v_{a\,int}}$$

$i_S v_{S_a}$: choose $= 0$ ; $i_L v_{L_a} = 0$ ; $i_{aS} v_S = 0$ as fix $v_a = 1$ & adjoint circuit doesn't change ; $i_{aL} v_L$: $-\Delta v_L$

cancel

$$-\Delta v_L = -\Delta\left(\frac{v_L}{v_S}\right) = v_{int}^T \Delta Y^T v_{a\,int}$$

$= -\Delta($voltage transfer function