For the previous example; "linear" part: 09/12/05

$$Ae^T v_t = \begin{bmatrix} sC_1 & 0 & 0 \\ 0 & sC_2+g_3 & 0 \\ 0 & 0 & g_1 \\ g_2 & -g_2 & -g_2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} i_4 \\ i_5 \\ i_6 \end{bmatrix} + \begin{bmatrix} i_s \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = BJ^T i_\ell - BJ$$ corrected

(here v_t & i_ℓ are for the full branches; subscript b left off for ease of writing)

nonlinear part: $K(v_5) = v_6 \Rightarrow K(v_2 - v_1) = v_1 - v_3$

Rewritten:

$$\begin{bmatrix} sC_1 & 0 & 0 & 1 & -1 & 1 \\ 0 & sC_2+g_3 & 0 & -1 & 1 & 0 \\ 0 & 0 & g_1 & -1 & 0 & -1 \\ g_2 & -g_2 & -g_2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$v_3 = v_1 - K(v_2 - v_1) \Rightarrow 0 = v_1 - v_3 - K(v_2 - v_1)$$

choose also $y = v_6 = v_1 - v_3$

Rewritten to isolate derivatives and get $E\dot{x} = a(x) + Bu$, $y = Cx$, x = semistate

$$s\begin{bmatrix} C_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} x = \begin{bmatrix} 0 & 0 & 0 & -1 & 1 & -1 \\ 0 & -g_3 & 0 & 1 & -1 & 0 \\ 0 & 0 & -g_1 & 1 & 0 & 1 \\ -g_2 & g_2 & g_2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -K(v_2 - v_1) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u \; ; \; x = \begin{bmatrix} v_t \\ \hline i_\ell \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}$$

$$y = [1 \; 0 \; -1 \; 0 \; 0 \; 0]\, x \qquad u = [i_s] \qquad y = [v_6]$$

For state equations need to eliminate 4 variables (normally keep capacitor voltages as the state variables) by solving the last 4 equations [note here easily $i_5 = 0$, $v_3 = v_1 - K(v_2 - v_1)$ then solve for $[i_4, i_6]^T = f\left(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\right)$]

---

close a finite circuit by a sphere

$p(t)$ into the sphere $\equiv 0$

if use $i_b(t)$, $v_b(t)$ then $p(t) = i_b^T(t) v_b(t) = 0$

assume $v_b = e^T v_t$; $i_b = \mathcal{J}^T i_\ell \Rightarrow p(t) = i_\ell^T \mathcal{J} \cdot e^T v_t = 0$

as for a generic graph can choose the v_{i_t} independent & same for i_{i_ℓ} (link current components)

$\therefore$ can cancel v_t & i_ℓ $\Rightarrow$ $\mathcal{J} \cdot \mathcal{C}^T = 0_{\ell \times t}$

if choose the same tree for forming $\mathcal{C}$ and $\mathcal{J}$; $\mathcal{C} = [1_t \,\vdots\, K]$, $\mathcal{J} = [K_\mathcal{J} \,\vdots\, 1_\ell]$

$$[K_\mathcal{J} \,\vdots\, 1_\ell]\begin{bmatrix} 1_t \\ \hdashline K^T \end{bmatrix} = 0_{\ell \times t} = K_\mathcal{J} \cdot 1_t + 1_\ell \cdot K^T$$

$$\Rightarrow -K_\mathcal{J} = K^T$$

$\therefore$ KVL $\Longleftrightarrow$ KCL

note $i_b^T(t_1) \cdot v_b(t_2) = 0$ even if $t_1 \neq t_2$ or $\underset{\text{circ } 1}{i_b^T} \cdot \underset{\text{circ } 2}{v_b} = 0$ if the two circuits have the same graph

(used to calculate sensitivities using a circuit that is the adjoint of another)

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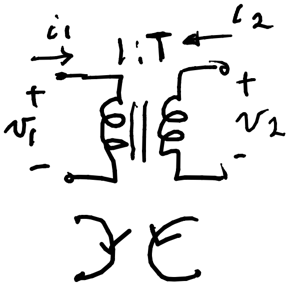
ideal transformer – 2-port



instantaneously lossless

$$P(t) = [i_1 \; i_2]\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$
$$= i_1 v_1 + i_2 v_2$$
$$= -T i_2 v_1 + i_2 v_2$$
$$\stackrel{!}{=} 0$$

sum of amp turns $= 0$

$$1 \cdot i_1 + T i_2 = 0$$
$$v_2 = T v_1$$

assume $i_2 \neq 0$

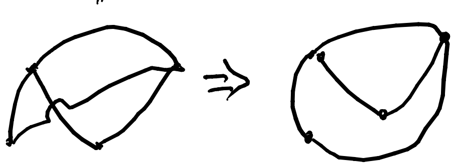
$$T v_1 = v_2$$

$$\begin{bmatrix} T & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & T \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

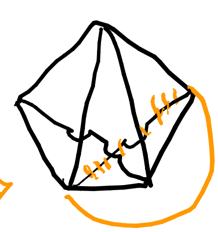
note this is $Av = Bi$
and $\det A = 0 = \det B$

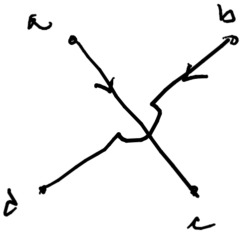
---

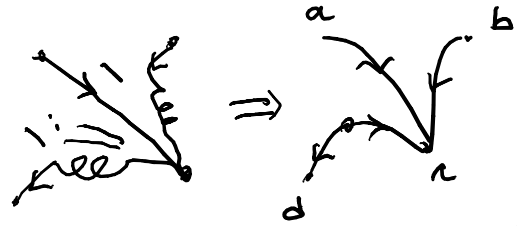
planar graph $\Rightarrow$ projection on a plane has no crossing branches



see fig. P 3.6 for a nonplanar



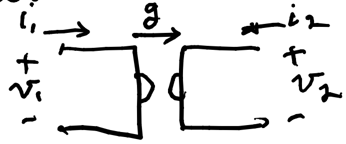




T = 1

allows a circuit to always have an equivalent with a planar graph

To work with admittances — use gyrators



g = gyration conductance

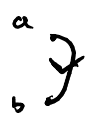
*

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & +g \\ -g & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow \begin{aligned} i_1 &= +g v_2 \\ i_2 &= -g v_1 \end{aligned}$$

$$\left.\begin{aligned} v_1 i_1 &= +g v_1 v_2 \\ v_2 i_2 &= -g v_1 v_2 \end{aligned}\right\} \quad p = [i_1 \; i_2]\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = -g v_1 v_2 + g v_1 v_2 \equiv 0$$

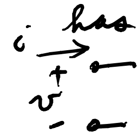
also instantaneously lossless

a short circuit has no admittance



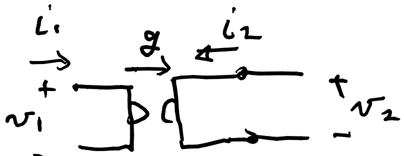
$v = 0$
i = arbitrary

an open circuit has an admittance



$i = 0$
v = arbitrary

here $Y = 0$ is its admittance



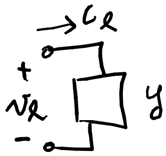
$i_2 = 0$
$= -g v_1 \Rightarrow v_1 = 0$ if $g \neq 0$

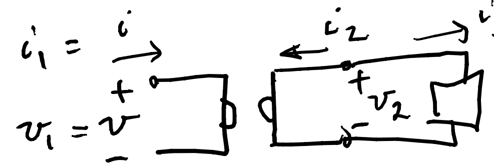
$\equiv$

a short

v_2 = arbitrary
$i_1 = -g v_2 \Rightarrow i_1$ = arbitrary if $g \neq 0$

if load with





$i_\ell = -i_2 = y_\ell v_2$

$v_\ell = v_2$

$i_1 = -g v_2$ $\qquad v_1 = -\frac{1}{g} i_2$

$$= -\frac{1}{g}(-y_\ell v_2) = +\frac{1}{g} y_\ell \left(+\frac{1}{g} i_1\right) = \frac{y_\ell}{g^2} i_1 = z_{in} i_1 = v_1$$

$$z_{in} = \frac{y_\ell}{g^2}$$