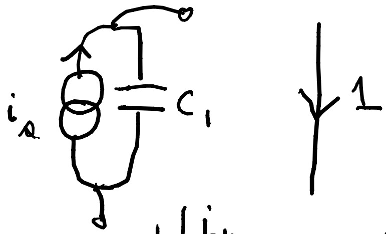
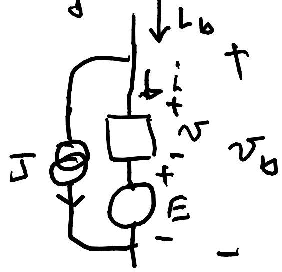
09/07/05



v_b = graph branch voltages

i_b = graph branch currents



$v_b = v + E$

$i_b = i + J$

our example as modified

$E = \underline{0}$ (a b-vector)

$$J = \begin{bmatrix} -i_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathcal{A} v = \mathcal{B} i$$
$$= \mathcal{B}(i_b - J)$$

$$\begin{bmatrix} sC_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & sC_2+g_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix}_b = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \left\{ \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}_b - \begin{bmatrix} -i_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

$K(v_5) = v_6$ op-amps voltage gain

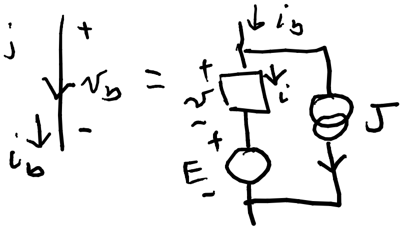
$v_b = \mathcal{C}^T v_t$, $i_b = \mathcal{D}^T i_\ell$

$$\begin{bmatrix} sC_1 & & & & \\ & sC_2+g_3 & & & \\ & & g_1 & & \\ & & & g_2 & \\ & & & & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \left\{ \begin{bmatrix} -1 & 1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_4 \\ i_5 \\ i_6 \end{bmatrix} + \begin{bmatrix} i_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

$K(v_2 - v_1) = v_1 - v_3$

General but linear, time invariant:

$v_b = e^T v_t \qquad i_b = \mathcal{J}^T i_\ell \; ; \qquad Av = Bi \; ;$



$v_b = v + E \, , \quad i_b = i + J$

$Av = A(v_b - E) = Bi = B(i_b - J)$

$= Ae^T v_t - AE = B\mathcal{J}^T i_\ell - BJ$

for b branches
A generally is $\hat{b} \times b$
B " " $\hat{b} \times b$

$(Ae^T v_t - B\mathcal{J}^T i_\ell) = AE - BJ$

$b = t + \ell$

$$\Rightarrow \left[Ae^T \;\vdots\; -B\mathcal{J}^T \right] \underbrace{\begin{bmatrix} v_t \\ i_\ell \end{bmatrix}}_{x \,=\, \text{unknowns} \,\to\, \text{a } b\text{-vector}} = \underbrace{AE - BJ}_{\text{sources (assume known)}}$$

if $\left[Ae^T \;\vdots\; -B\mathcal{J}^T \right]^{-1}$ exists, can solve for x

$$x = \left[Ae^T \;\vdots\; -B\mathcal{J}^T \right]^{-1} \left[A \;\vdots\; -B \right] \underbrace{\begin{bmatrix} E \\ J \end{bmatrix}}_{\text{input } u}$$

if isolate capacitors into the tree
with loops of capacitors

$$\begin{bmatrix} C_{cap} & 0 \\ 0 & 0 \end{bmatrix} \dot{x} = \Big[A_{\substack{const \\ elem}} \Big] x + \Big[B_{coef.} \Big] u$$

$$y = \left[C_{out} \right] x$$

Ex:

$$\begin{bmatrix} c_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \dot{v}_3 \\ \dot{i}_4 \\ \dot{i}_5 \\ \dot{i}_6 \end{bmatrix} = \Big[\qquad \qquad \Big] x + \Big[\qquad \qquad \Big] \big[i_R \big]$$

Call x the semistate $= \begin{bmatrix} v_t \\ i_\ell \end{bmatrix}$ as an example generally $E\frac{dx}{dt} = A(x,t) + Bu$ assumes constant linear capacitors

$y = Cx$

if linear and time invariant $A(x,t) = A \cdot x$ ← constant matrix

$$sEx = E\frac{dx}{dt} = Ax + Bu$$

$$y = Cx$$

$\Rightarrow$ $(sE - A)x = Bu$; $(sE-A)^{-1}Bu = x$ if inverse exists

$$y = \{C(sE-A)^{-1}B\}u \Rightarrow T(s) = C(sE-A)^{-1}B$$

if E = identity then at $s=\infty$, $T(s) = 0$

But if $E \neq 1_n$ then $T(s) = s$ is possible

Ex: $s\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}x = \begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}x + \begin{bmatrix}0\\1\end{bmatrix}u$

$y = [0 \quad 1]x$

$(sE-A) = \begin{bmatrix}s & -1\\1 & 0\end{bmatrix}$

$(sE-A)^{-1} = \frac{1}{1}\begin{bmatrix}0 & 1\\-1 & s\end{bmatrix}$

$$T(s) = [0 \quad 1]\begin{bmatrix}0 & 1\\-1 & s\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = s$$

Ex: $s\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}x = \begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}x + \begin{bmatrix}0\\1\end{bmatrix}u$

$y = [0 \quad 1]x$

$sE - A = \begin{bmatrix}s & -1\\1 & s\end{bmatrix}$

$(sE-A)^{-1} = \frac{1}{s^2+1}\begin{bmatrix}s & 1\\-1 & s\end{bmatrix}$

$$T(s) = \frac{s}{s^2+1} \rightarrow T(\infty) = 0$$