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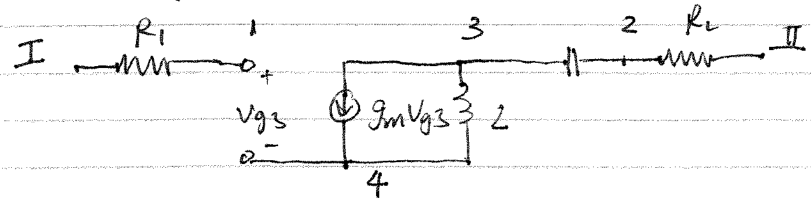
Problem 1,

$$I_d = \frac{kp}{2}\frac{W}{L}(V_{gs}-V_{th})^2$$

$$V_{th} = 1\,v, \quad kp = 10^{-5} A/V^2 \quad w/l = 10$$

10' a): small signal equivalent circuit:



$$\boxed{g_m = kp\frac{W}{L}(V_{gs}-V_{th})}$$

10' b) Yind matrix between node 1, 2 and ground.

$$I_1 = 0$$

$$I_2 = (V_2 - V_3)\cdot SC = SCV_2 - SCV_3$$

$$I_3 = g_m V_{gs} + \frac{1}{SL}V_{34} + SC\cdot V_{32}$$

$$I_3 = g_m V_1 - SCV_2 + \left(\frac{1}{SL} + SC\right)V_3 - \left(g_m + \frac{1}{SL}\right)V_4$$

$$I_4 = -g_m V_{gs} + \frac{1}{SL}V_{43} = -g_m V_1 - \frac{1}{SL}V_3 + \left(g_m + \frac{1}{SL}\right)V_4$$

$$\therefore \boxed{Y_{ind} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & SC & -SC & 0 \\ g_m & -CS & \frac{1}{LS}+CS & -(g_m+\frac{1}{LS}) \\ -g_m & 0 & -\frac{1}{LS} & g_m+\frac{1}{LS} \end{bmatrix}} \qquad \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = Y_{ind}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

12' c) ground 4 to ground $V_4 = 0$

$$Y_{def} = \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & SC & -SC \\ g_m & -SC & \frac{1}{SL}+SC \end{array}\right] \qquad \begin{bmatrix} I_1 \\ I_2 \\ \hdashline I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$\begin{cases} Y_{11} = \begin{bmatrix} 0 & 0 \\ 0 & SC \end{bmatrix} \\ Y_{12} = \begin{bmatrix} 0 \\ -SC \end{bmatrix} \\ Y_{21} = \begin{bmatrix} g_m & -SC \end{bmatrix} \\ Y_{22} = \frac{1}{SL} + SC \end{cases}$$

$$I_3 = 0$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = Y\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad Y = Y_{11} - Y_{12}Y_{22}^{-1}Y_{21} = \begin{bmatrix} 0 & 0 \\ 0 & SC \end{bmatrix} - \begin{bmatrix} 0 \\ -SC \end{bmatrix}\cdot\frac{1}{(\frac{1}{SL}+SC)}\cdot\begin{bmatrix} g_m & -SC \end{bmatrix}$$

$$\therefore Y = \begin{bmatrix} 0 & 0 \\ \frac{S^2Lcg_m}{1+S^2Lc} & \frac{SC}{1+S^2Lc} \end{bmatrix}$$

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Problem 1 (continues)

12' d) $R_1 = R_2 = 1$ Yaug-ind for the two ports between node I and II.

$$I_I = (V_I - V_1)/R_1 = V_I - V_1$$

$$I_{II} = (V_{II} - V_2)/R_2 = V_{II} - V_2$$

$$I_1 = (V_1 - V_I)/R_1 = -V_I + V_1$$

$$I_2 = (V_2 - V_{II})/R_2 + (V_2 - V_3)\cdot sC = -V_{II} + (1+sC)V_2 - sCV_3$$

$$I_3 = g_m V_1 - sCV_2 + \left(\frac{1}{sL} + sC\right)V_3 - \left(g_m + \frac{1}{sL}\right)V_4$$

$$I_4 = -g_m V_1 - \frac{1}{sL}V_3 + \left(g_m + \frac{1}{sL}\right)V_4$$

$$\boxed{Y_{aug\text{-}ind} = \begin{array}{c} \begin{array}{cccccc} \mathrm{I} & \mathrm{II} & 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1+sC & -sC & 0 \\ 0 & 0 & g_m & -sC & sC+\frac{1}{Ls} & -\left(g_m+\frac{1}{sL}\right) \\ 0 & 0 & -g_m & 0 & -\frac{1}{sL} & g_m+\frac{1}{sL} \end{bmatrix} \end{array}}$$

12' e) ground 4, $V_4 = 0$

$$Y_{aug\text{-}def} = \left[\begin{array}{cc|ccc} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ \hline -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1+sC & -sC \\ 0 & 0 & g_m & -sC & sC+\frac{1}{Ls} \end{array}\right] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$\begin{bmatrix} I_I \\ I_{II} \\ \hline I_1 \\ I_2 \\ I_3 \end{bmatrix} = Y_{aug\text{-}def} \cdot \begin{bmatrix} V_I \\ V_{II} \\ V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$\begin{bmatrix} I_I \\ I_{II} \end{bmatrix} = Y_{aug} \cdot \begin{bmatrix} V_I \\ V_{II} \end{bmatrix}$$

$$Y_{aug} = Y_{11} - Y_{12}Y_{22}^{-1}Y_{21} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1+sC & -sC \\ g_m & -sC & sC+\frac{1}{sL} \end{bmatrix}^{-1} \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}$$

$$\therefore \boxed{Y_{aug} = \begin{bmatrix} 0 & 0 \\ \dfrac{s^2LCg_m}{1+sC+s^2LC} & \dfrac{sL}{1+sC+s^2LC} \end{bmatrix}}$$

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Problem 1. (continuous)

12' f) $Y = \begin{bmatrix} 0 & 0 \\ \frac{s^2 L c g_m}{1+s^2 L C} & \frac{sc}{1+s^2 LC} \end{bmatrix}$ $\quad Y_{aug} = \begin{bmatrix} 0 & 0 \\ \frac{s^2 L c g_m}{1+sc+s^2 LC} & \frac{sc}{1+sc+s^2 LC} \end{bmatrix}$

Method 1) $S(s) = (1+R^{-1}Y)^{-1}(1-R^{-1}Y)$

$\therefore S(s) = \begin{bmatrix} 1 & 0 \\ \frac{s^2 L c g_m}{1+s^2 L C} & \frac{1+sc+s^2 LC}{1+s^2 LC} \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 & 0 \\ -\frac{s^2 L c g_m}{1+s^2 LC} & \frac{1-sc+s^2 LC}{1+s^2 LC} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{-2 s^2 L c g_m}{1+sc+s^2 LC} & \frac{1-sc+s^2 LC}{1+sc+s^2 LC} \end{bmatrix}$

Method 2) $S(s) = 1 - 2Y_{aug}$

$$\therefore \boxed{S(s) = \begin{bmatrix} 1 & 0 \\ \frac{-2 s^2 L c g_m}{1+sc+s^2 LC} & \frac{1-sc+s^2 LC}{1+sc+s^2 LC} \end{bmatrix}}$$

clearly, two methods agree.

12' g) $S(s)$ is bounded-real means:

$x^{T*}(I_2 - S^{T*}(s)\, S(s))\, x \geq 0$ in $Re\, s \geq 0$

for $s = j\omega$

$1_2 - S^{T*}(j\omega)S(j\omega) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & \frac{2\omega^2 L c g_m}{(1-\omega^2 LC) - j\omega L} \\ 0 & e^{-j\theta} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{2\omega^2 L c g_m}{(1-\omega^2 LC) + j\omega L} & e^{j\theta} \end{bmatrix}$

$e^{j\theta} = \frac{(1-\omega^2 LC) - j\omega c}{(1-\omega^2 LC) + j\omega C}$. $\quad b = 2\omega^2 L c g_m / [(1-\omega^2 LC) + j\omega L]$

then $1_2 - S^{T*}(j\omega)\, S(j\omega) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & b^* \\ 0 & e^{-j\theta} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ b & e^{j\theta} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1+|b|^2 & b^* e^{j\theta} \\ b e^{-j\theta} & 1 \end{bmatrix}$

$\therefore 1_2 - S^{T*}(j\omega)\, S(j\omega) = \begin{bmatrix} -|b|^2 & b^* e^{j\theta} \\ b e^{-j\theta} & 0 \end{bmatrix}$

$\det\left(1_2 - S^{T*}(j\omega) S(j\omega)\right) = -|b|^2 \leq 0$

so. for any $b \neq 0$, $S(s)$ is not bounded-real

to make $S(s)$ bounded real, $\boxed{b = 0 \text{ , } L = 0 \text{ or } C = 0}$

$L = 0 \quad S(s) = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1-sL}{1+sL} \end{bmatrix}$ $\quad$ or $C = 0 \quad S(s) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

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Problem 2:

P(s) has no zeros at Re s > 0, ⇒ P(s) has Hurwitz property

P(s) = ev(s) + odd(s)

if $Z(s) = \frac{ev(s)}{odd(s)}$ $\left(\text{or } \frac{odd(s)}{even(s)}\right)$ is lossless, which means it can be synthesized with LC elements, the P(s) has Hurwitz property

10' a) $P_1(s) = s^6 + 6s^5 + 5s^4 + 4s^3 + 3s^2 + 2s + 1$ $\qquad Z(s) = \frac{s^6+5s^4+3s^2+1}{6s^5+4s^3+2s}$

1st cauer:

$$
\begin{array}{l}
6s^5+4s^3+2s \;\overline{)\; s^6+5s^4+3s^2+1} \quad \frac{1}{6}s \\
\quad s^6+\frac{2}{3}s^4+\frac{1}{3}s^2 \\
\quad \frac{13}{3}s^4+\frac{8}{3}s^2+1 \;\overline{)\; 6s^5+4s^3+2s} \quad \frac{18}{13}s \\
\quad\quad 6s^5+\frac{48}{13}s^3+\frac{18}{13}s \\
\quad\quad \frac{4}{13}s^3+\frac{8}{13}s \;\overline{)\; \frac{13}{3}s^4+\frac{8}{3}s^2+1} \quad \frac{169}{12}s \\
\quad\quad\quad \frac{13}{3}s^4+\frac{26}{3}s^2 \\
\quad\quad\quad -\frac{18}{3}s^2+1 \;\overline{)\; \frac{4}{13}s^3+\frac{8}{13}s} \quad -\frac{2}{39}s \\
\quad\quad\quad\quad \frac{4}{13}s^3-\frac{2}{39}s \\
\quad\quad\quad\quad \frac{2}{3}s \;\overline{)\; -6s^2+1} \quad -9s \\
\quad\quad\quad\quad\quad -6s^2 \\
\quad\quad\quad\quad\quad 1 \;\overline{)\; \frac{2}{3}s} \quad \frac{2}{3}s \\
\quad\quad\quad\quad\quad\quad \frac{2}{3}s \\
\quad\quad\quad\quad\quad\quad 0
\end{array}
$$

$$\therefore Z(s) = \frac{1}{6}s + \cfrac{1}{\frac{18}{13}s + \cfrac{1}{\frac{169}{12}s + \cfrac{1}{\left(-\frac{2}{39}\right)s + \cfrac{1}{(-9)s + \cfrac{1}{\frac{2}{3}s}}}}}$$

Z(s) not synthesizable with LC, $P_1(s)$ doesn't have Hurwitz property

10' b) $P_2 = s^5 + 2s^4 + 3s^4 + 4s^2 + 5$ $\qquad Z(s) = \frac{s^5+3s^3+0s}{2s^4+4s^2+5}$

1st cauer:

$$Z(s) = \frac{1}{2}s + \cfrac{1}{2s + \cfrac{1}{\frac{1}{9}s + \cfrac{1}{-\frac{162}{55}s + \cfrac{1}{-\frac{11}{8}s}}}}$$

$P_2(s)$ doesn't have Hurwitz property