ENEE 610 HW#4 10/25/04

Problem 2: $Z(s) = \dfrac{s^2(s^2+b)}{(s+a)(s^2+c)}$ lossless positive real

a) All poles and zeros are simple and on the imaginary axis
poles and zeros alternate.

10'

$$\begin{cases} a=0 \\ 0 \le c \le b \\ \text{for } c>0,\ b>c \end{cases} \qquad Z(s) = \frac{s(s^2+b)}{(s^2+c)} \qquad 3' \quad b>c$$

special case $c=0,\ b\neq 0$ $\quad Z(s) = \dfrac{s^2+b}{s}$

$c=0=b \quad Z(s) = s$

5' b) for case $a=0, \quad 0<c<b$

$$Z(s) = \frac{s(s^2+b)}{s^2+c}$$

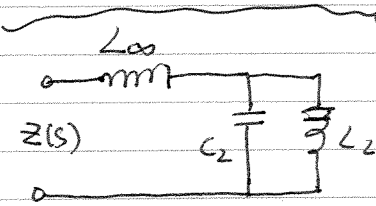
1st Foster's

3'

$$Z(s) = k_\infty s + \frac{2k_2 s}{s^2+\omega_2^2} \qquad \omega_2^2 = c$$

$$k_\infty = \left.\frac{Z(s)}{s}\right|_{s\to\infty} = 1$$

$$2k_2 = \left.\frac{s^2+c}{s}\cdot Z(s)\right|_{s^2=-c} = b-c$$



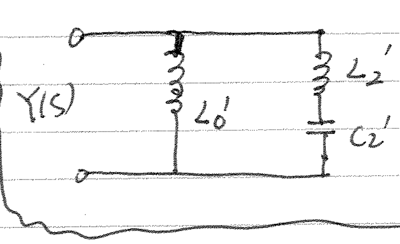
$$\begin{cases} L_\infty = k_\infty = 1 \\ C_2 = \dfrac{1}{2k_2} = \dfrac{1}{b-c} \\ L_2 = \dfrac{2k_2}{\omega_2^2} = \dfrac{b-c}{c} \end{cases}$$

5' 2nd Foster's

$$Y(s) = \frac{s^2+c}{s(s^2+b)} = k_0'\frac{1}{s} + \frac{2k_2' s}{s^2+\omega_2'^2} \qquad \omega_1^2 = b$$

$$k_0' = \left.Y(s)\cdot s\right|_{s=0} = \frac{c}{b}$$

$$2k_2' = \left.\frac{s^2+\omega_2'^2}{s}\cdot Y(s)\right|_{s^2=-\omega_2'^2=-b} = \frac{b-c}{b}$$



$$\begin{cases} L_0' = \dfrac{1}{k_0'} = \dfrac{b}{c} \\ L_2' = \dfrac{1}{2k_2'} = \dfrac{b}{b-c} \\ C_2' = \dfrac{2k_2'}{\omega_2'^2} = \dfrac{b-c}{b^2} \end{cases}$$

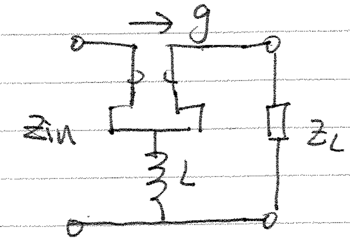
ENEE610 HW#4 10/25/04

Problem 2

b) (continuous)

S^1 Richard's function: $R(s) = \dfrac{k\,z(s) - s\,z(k)}{k\,z(k) - s\,z(s)}$ (PP. 361 from text book) ①

$$R(s) = \left[\frac{s}{k} - \frac{z(s)}{z(k)}\right] \Big/ \left[\frac{s}{k}\cdot\frac{z(s)}{z(k)} - 1\right] \quad ②$$



$$Z_L = \frac{\frac{1}{g^2} - z_{in}(s) L s}{z_{in}(s) - sL} = \frac{1}{g^2 z_{in}(k)} \cdot \frac{L g^2 \cdot z_{in}(s)\cdot s - 1}{\frac{sL}{z_{in}(k)} - \frac{z_{in}(s)}{z_{in}(k)}} \quad ③$$

compare ② & ③

$$\begin{cases} k\,z(k) = 1/Lg^2 \\ k = z_{in}(k)/L \\ Z_L = \dfrac{1}{g^2 z_{in}(k)} \cdot \dfrac{1}{Rich(z_{in}(s))} \end{cases} \Rightarrow \boxed{\begin{cases} L = z_{in}(k)/k \\ g = \pm\, 1/z_{in}(k) \\ Z_L = z_{in}(k) / Rich(z_{in}(s)) \end{cases}}$$

to reduce the degree, find zero of the even part

$z_{in}(k) + z_{in}(-k) = 0$ $z_{in}(s)$ is odd function, any number works

choose $k=1$, $z_{in}(s) = \dfrac{s(s^2+b)}{s^2+c}$ $z_{in}(k) = \dfrac{b+1}{c+1}$

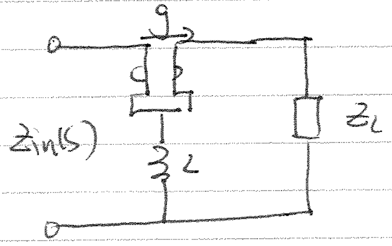
$$k\,z_{in}(s) - s\cdot z_{in}(k) = -\frac{1}{(s^2+c)(c+1)}\cdot (b-c)\cdot s(s^2-1)$$

$$k\,z_{in}(k) - s\,z_{in}(s) = -\frac{1}{(s^2+c)(c+1)}\left[(c+1)s^2 + (bc+c)\right]\cdot(s^2-1)$$

$$\therefore Rich(z_{in}(s)) = \frac{(b-c)s}{(c+1)s^2 + (bc+c)}$$

$$Z_L = z_{in}(k) / Rich(z_{in}(s)) = \frac{b+1}{c+1}\cdot\frac{(c+1)s^2 + (bc+c)}{(b-c)s}$$ 1 degree less than z_{in}

$$\therefore \boxed{\begin{cases} L = \dfrac{z_{in}(k)}{k} = \dfrac{b+1}{c+1} \\ g = \pm\, 1/z_{in}(k) = \pm\,\dfrac{c+1}{b+1} \\ Z_L = \dfrac{b+1}{c+1}\cdot\dfrac{(c+1)s^2 + (bc+c)}{(b-c)s} \end{cases}}$$



ENEE 610 HW#4 10/25/04

Problem 3

$$m\frac{d^2x}{dt^2} = -b\frac{dx}{dt} - k(L_0 - x) + \frac{q^2}{2A\varepsilon}$$

$$\left\{\begin{array}{l} k = 3.9\times10^{-9}\,N/m \\ b = 1.1\times10^{-15}\,N\text{-}s/m \\ A = (100\,\mu m)^2 = 10^{-8}\,m^2 \\ \varepsilon = 8.85\times10^{-12}\,F/m \end{array}\right.$$

$V = r\cdot i + V_c$, $q = CV_c$, $C = \frac{\varepsilon A}{L_0 - x}$, $i = \frac{dq}{dt}$

a) steady state $\frac{dx}{dt} = 0$, $\frac{d^2x}{dt^2} = 0$, also, $x = 0$

10'

$$-kL_0 + \frac{\bar{q}^2}{2A\varepsilon} = 0 \quad \text{------} \quad ①$$

$$i = \frac{dq}{dt} = 0 \quad \therefore V = V_c, \quad \bar{q} = CV_c = CV = \frac{\varepsilon A}{L_0}V \quad ②$$

from ① ② solve $L_0 = \sqrt[3]{\frac{\varepsilon A V^2}{2k}} = 3.57\times10^{-4}\,M$

$$\bar{q} = \frac{\varepsilon A V}{L_0} = 4.96\times10^{-16}\,C$$

b) $X = [x, \dot{x}, q]^T$ find state equation, $x_1 = x$, $x_2 = \dot{x}$, $x_3 = q$.

$$\frac{dx}{dt} = \dot{x} \quad \text{------} \quad ④$$

25'

$$m\frac{d^2x}{dt^2} = -b\dot{x} + kx + \frac{q^2}{2A\varepsilon} - kL_0 \quad \text{---} \quad ⑤$$

$$V = r\cdot i + V_c = r\cdot\frac{dq}{dt} + \frac{q(L_0 - x)}{\varepsilon A}$$

$$\therefore r\frac{dq}{dt} = -\frac{L_0}{\varepsilon A}q + \frac{qx}{\varepsilon A} + V \quad \text{-----} \quad ⑥$$

From ④, ⑤ and ⑥, with $u = V$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & r \end{bmatrix} \frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \\ q \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ k & -b & 0 \\ 0 & 0 & -\frac{L_0}{\varepsilon A} \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \\ q \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{q^2}{2\varepsilon A} - kL_0 \\ \frac{qx}{\varepsilon A} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} V$$

15'

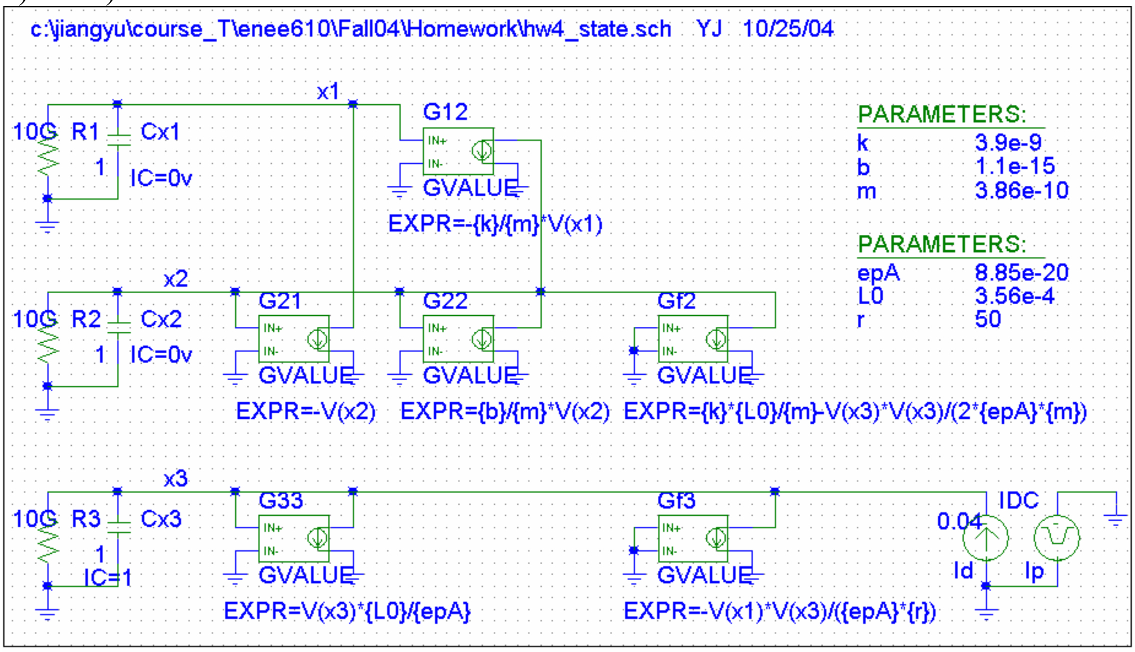
or

$$\boxed{\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{k}{m} & -\frac{b}{m} & 0 \\ 0 & 0 & -\frac{L_0}{\varepsilon A r} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{x_3^2}{2\varepsilon A m} - \frac{kL_0}{m} \\ \frac{x_1 x_3}{\varepsilon A r} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{r} \end{bmatrix} V}$$

$$m = 19.3\times10^3\,kg/M^3 \cdot 10^{-8}\,M^2 \cdot 2\times10^{-6}\,M = 3.86\times10^{-10}\,kg$$

c) 5'

b) and c)



For Gf2, since 1/(2*{epA}*{m})=1.46e28, every large number, Gf2 fail to converge in simulation.

10 Messages: 8 Error, 0 Warning, 2 Info
Severity Origin Time Message Text
INFO Schematics 04:11PM Creating PSPICE netlist...
INFO Schematics 04:11PM Netlist completed.
ERROR PSpiceAD 04:11PM Convergence problem in bias point calculation
ERROR PSpiceAD 04:11PM These devices failed to converge:
ERROR PSpiceAD 04:11PM G_Gf2
ERROR PSpiceAD 04:11PM Discontinuing simulation due to convergence problem
ERROR PSpiceAD 04:11PM Convergence problem in transient bias point calculation
ERROR PSpiceAD 04:11PM These devices failed to converge:
ERROR PSpiceAD 04:11PM G_Gf2
ERROR PSpiceAD 04:11PM Discontinuing simulation due to convergence problem

One surgestion of solving this problem is to normalize x3 to Qdc.

As shown in part d) later, this system has pole at real positive axis. Therefore, it is not stable.

EUEE 610 HW#4 10/25/04

Problem 3 (continues)

10

d) $$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ k/m & -b/m & 0 \\ 0 & 0 & -L_0/\varepsilon A r \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{x_3^2}{2\varepsilon A m} - \frac{kL_0}{m} \\ \frac{x_1 x_3}{\varepsilon A r} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{r} \end{bmatrix} V \quad \cdots (1)$$

at DC operating point
$$\begin{cases} \bar{x}_1 = \bar{x} = 0 \\ \bar{x}_2 = \dot{\bar{x}}_1 = 0 \\ \bar{x}_3 = \bar{q} = \frac{\varepsilon A V_{dc}}{L_0} = 4.96 \times 10^{-16} \end{cases} \qquad \begin{cases} x_1 = \bar{x}_1 + \hat{x}_1 \\ x_2 = \bar{x}_2 + \hat{x}_2 \\ x_3 = \bar{x}_3 + \hat{x}_3 \end{cases} \quad \text{also } V = V_{dc} + \hat{v}$$

equation (1) becomes:

$$\frac{d}{dt}\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{k}{m} & -\frac{b}{m} & 0 \\ 0 & 0 & -L_0/\varepsilon A r \end{bmatrix}\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \bar{x}_3 + \hat{x}_3 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{(\bar{x}_3 + \hat{x}_3)^2}{2\varepsilon A m} - \frac{kL_0}{m} \\ \frac{\hat{x}_1(\bar{x}_3 + \hat{x}_3)}{\varepsilon A r} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{r} \end{bmatrix}(V_{dc} + \hat{v})$$ since $\bar{x}_1 = 0$ and $\bar{x}_2 = 0$

only keep the first order term

$$f(x) = \begin{bmatrix} 0 \\ \left(\cancel{\frac{\bar{x}_3^2}{2\varepsilon A m} - \frac{kL_0}{m}}\right) + \frac{\bar{x}_3}{\varepsilon A m}\hat{x}_3 \\ \frac{\bar{x}_3}{\varepsilon A r}\hat{x}_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{\bar{x}_3}{\varepsilon A m} \\ \frac{\bar{x}_3}{\varepsilon A r} & 0 & 0 \end{bmatrix}\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix}$$

since $\bar{x}_3^2 = \bar{q}^2 = 2\varepsilon A \cdot kL_0$

$$\therefore \frac{d}{dt}\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{k}{m} & -\frac{b}{m} & \frac{\bar{x}_3}{\varepsilon A m} \\ \frac{\bar{x}_3}{\varepsilon A r} & 0 & -\frac{L_0}{\varepsilon A r} \end{bmatrix}\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \left(\cancel{-\frac{\bar{x}_3 L_0}{\varepsilon A r} + \frac{V_{dc}}{r}}\right) + \frac{\hat{v}}{r} \end{bmatrix}$$

since $\bar{x}_3 = \bar{q} = \frac{\varepsilon A V_{dc}}{L_0}$

$$\begin{bmatrix} s & -1 & 0 \\ -\frac{k}{m} & s + \frac{b}{m} & -\frac{\bar{x}_3}{\varepsilon A m} \\ -\frac{\bar{x}_3}{\varepsilon A r} & 0 & s + \frac{L_0}{\varepsilon A r} \end{bmatrix}\begin{bmatrix} \hat{x}_1(s) \\ \hat{x}_2(s) \\ \hat{x}_3(s) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{\hat{V}(s)}{r} \end{bmatrix}$$

$$\hat{x}_1(s) = \varepsilon A \hat{V}(s) \Big/ \left\{ s^3 \varepsilon^2 A^2 m r + s^2(\varepsilon A m L_0 + \varepsilon^2 A^2 b r) + s(\varepsilon A b L_0 - k\varepsilon^2 A^2 r) - (\bar{q}^2 + k\varepsilon A L) \right\}$$

$$\therefore \boxed{\frac{\hat{x}_1(s)}{\hat{V}(s)} = \varepsilon A \Big/ \left\{ s^3 \varepsilon^2 A^2 m r + s^2(\varepsilon A m L_0 + \varepsilon^2 A^2 b r) + s(\varepsilon A b L_0 - k\varepsilon^2 A^2 r) - \bar{q}^2 + k\varepsilon A L \right\}}$$

pole at $\sigma > 0$ plane