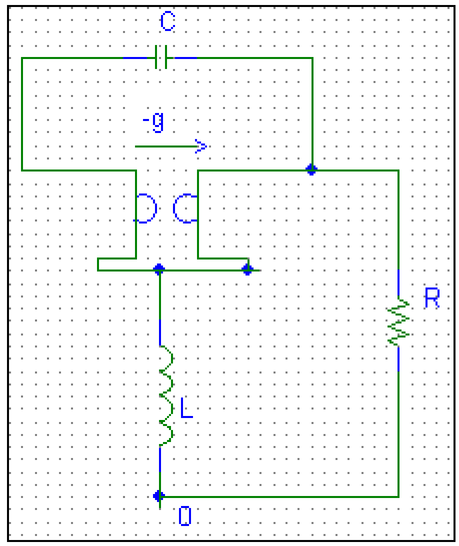
file: c:\jiangyu\Courses_T\enee610\Fall04\HomeWork\HW3_1.mcd
HW5: Problem1 solution JY 10/11/04

a) (5)

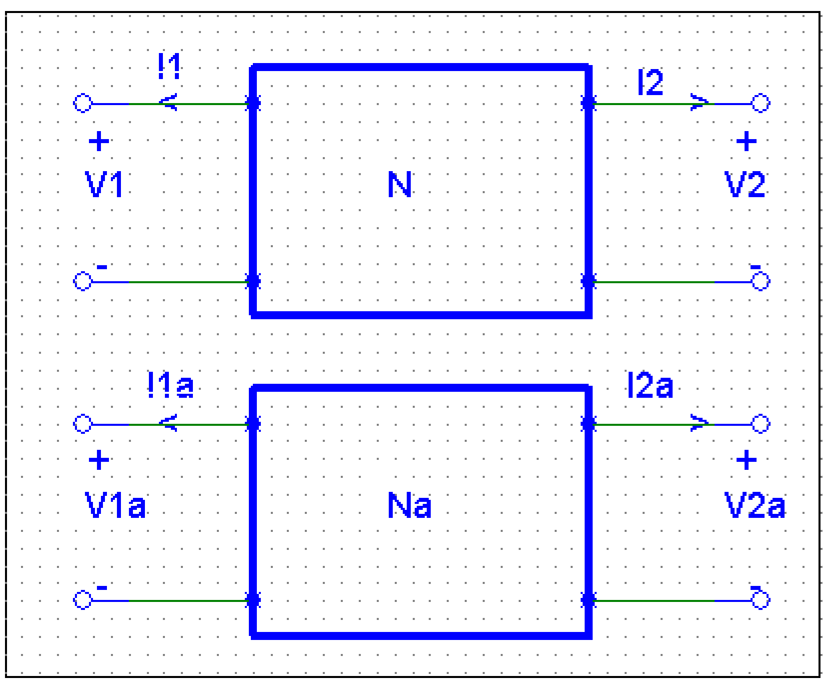
The adjoint circuit:



b) (10)

The key point of calculating the sensitivity is to find how to terminated the original and adjoint circuit.

In this case, yin(s)=Iin(s)/Vin(s):

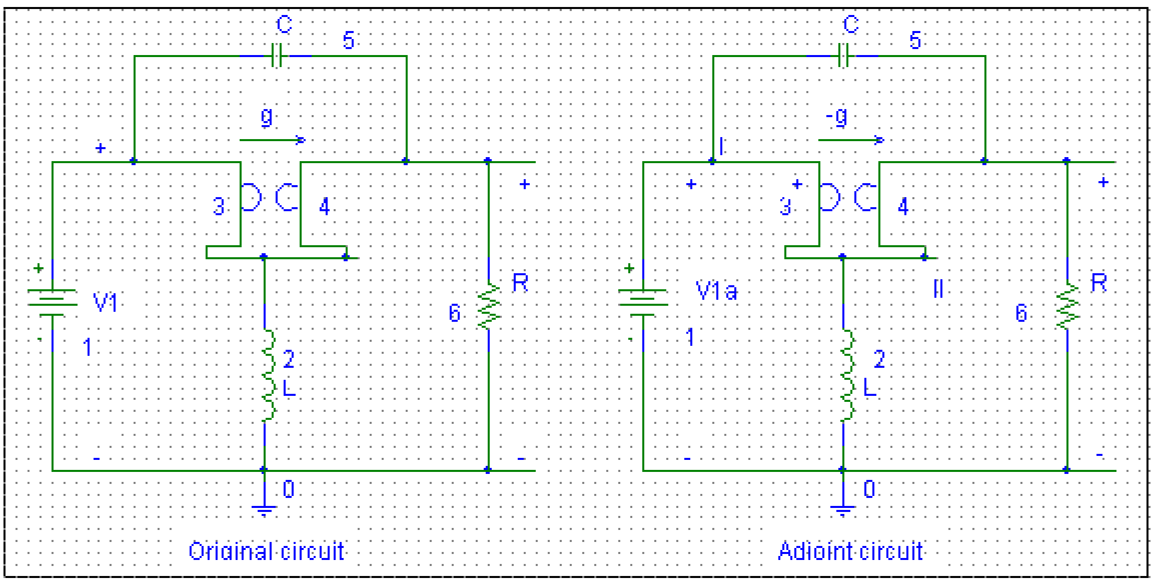


We have to start with: realizing that all the voltage signs and current directions are show above.

$V_1 * i_1^a + V_2 * i_2^a - V_1^a * i_1 - V_2^a * i_2 = -[i_N^{aT} * V_N - i_N^T * V_N^a]$

To find sensitivity of $Y(s) = -i_1/V_1$, The original and adjoint circuits are terminated in a way that sets $V_1 = V_1^a = 1$, and $i_2 = i_2^a = 0$. There for, $Y(s) = -i_1$, and $dY(s)/da = -di_1/da$.

The original and adjoint circuits are terminated as shown below.



Clearly $i_2 = i_2^a = 0$

So $V_1 * i_1^a - V_1^a * i_1 = -[i_N^{aT} * V_N - i_N^T * V_N^a]$

Since is V_1 fixed in a, and nothing varies in N^a,

then $dV_1/da = 0$, $dV_1^a/da = 0$, and $di_1^a/da = 0$,

So $-V_1^a * di_1/da = -[i_N^{aT} * V_N - i_N^T * V_N^a]$

Then $di_1/da = d[i_N^{aT} * V_N - i_N^T * V_N^a]/da/V_1^a = [V_N^{aT} * (dY_N/da) * V_N]/V_1^a$

$Y(s) = -i_1/V_1$

$dY(s)/da = -di_1/da = -[V_N^{aT} * (dY_N/da) * V_N]$ by setting $V_1 = V_1^a = 1$

$ST(a) = -(a/i_1) * (di_1/da)$

Circuit annalysis of the original and adjoint circuit:

They are have the same gragh.

Choose 2, 3, 4 to be the tree branch.

Cut set Matrix:

$$C := \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 \end{bmatrix}$$

Tie set Matrix:

$$T := \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \end{bmatrix}$$

Using equality $-C*j=-C*Ybb*C^T*Vt$

$$j := (i1ORi1a \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0)^T$$

$$Left := -C \cdot j$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} i1ORi1a \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot (-1) \qquad -C*j= \begin{bmatrix} -i1OR - i1a \\ -i1OR - i1a \\ 0 \end{bmatrix}$$

For orinal circuit, I1 to make V1=1, I2a=0

Admittance matrix:

$$Ybb := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{s \cdot L} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g & 0 & 0 \\ 0 & 0 & -g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & s \cdot C & 0 \\ 0 & 0 & 0 & 0 & 0 & R \end{bmatrix}$$

Evaluate CYbbCT=C*Ybb*C^T

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{s\cdot L} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g & 0 & 0 \\ 0 & 0 & -g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & s\cdot C & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{R} \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$$

CYbbCT=

$$\begin{bmatrix} \frac{1}{(s\cdot L)} + \frac{1}{R} & 0 & \frac{1}{R} \\ 0 & s\cdot C & g - s\cdot C \\ \frac{1}{R} & -g - s\cdot C & s\cdot C + \frac{1}{R} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{(s\cdot L)} + \frac{1}{R} & 0 & \frac{1}{R} \\ 0 & s\cdot C & g - s\cdot C \\ \frac{1}{R} & -g - s\cdot C & s\cdot C + \frac{1}{R} \end{bmatrix} \cdot \begin{bmatrix} V1 \\ V2 \\ V3 \end{bmatrix} = \begin{bmatrix} -i1 \\ -i1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} V2 \\ V3 \\ V4 \end{bmatrix} := \begin{bmatrix} \frac{s\cdot(L\cdot R\cdot g^2 - L\cdot g)}{(s\cdot(C + L\cdot g^2) + R\cdot g^2)} \\ \frac{s\cdot(R\cdot C + L\cdot g) + 1}{(s\cdot(C + L\cdot g^2) + R\cdot g^2)} \\ \frac{s\cdot(R\cdot C + L\cdot g) + R\cdot g}{(s\cdot(C + L\cdot g^2) + R\cdot g^2)} \end{bmatrix} \cdot (-i1)$$

$$VN := \begin{bmatrix} \frac{s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1}{(s\cdot(C + L\cdot g^2) + R\cdot g^2)} \\ \frac{s\cdot(L\cdot R\cdot g^2 - L\cdot g)}{(s\cdot(C + L\cdot g^2) + R\cdot g^2)} \\ \frac{s\cdot(R\cdot C + L\cdot g) + 1}{(s\cdot(C + L\cdot g^2) + R\cdot g^2)} \\ \frac{s\cdot(R\cdot C + L\cdot g) + R\cdot g}{(s\cdot(C + L\cdot g^2) + R\cdot g^2)} \\ \frac{1 - R\cdot g}{(s\cdot(C + L\cdot g^2) + R\cdot g^2)} \\ \frac{s\cdot(L\cdot R\cdot g^2 + R\cdot C) + R\cdot g}{(s\cdot(C + L\cdot g^2) + R\cdot g^2)} \end{bmatrix} \cdot (-i1)$$

Since V1=1

$$i1 := \frac{-(s\cdot(C + L\cdot g^2) + R\cdot g^2)}{s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1}$$

$$Yin(s) := \frac{s\cdot(C + L\cdot g^2) + R\cdot g^2}{s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1}$$

$$VN := \begin{bmatrix} 1 \\ \dfrac{s\cdot(L\cdot R\cdot g^2 - L\cdot g)}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \\ \dfrac{s\cdot(R\cdot C + L\cdot g) + 1}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \\ \dfrac{s\cdot(R\cdot C + L\cdot g) + R\cdot g}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \\ \dfrac{1 - R\cdot g}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \\ \dfrac{s\cdot(L\cdot R\cdot g^2 + R\cdot C) + R\cdot g}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \end{bmatrix}$$

For adjoint circuit, I1a to make V2a=1, I2a=0

Admittance matrix :

$$Ybba := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \dfrac{1}{s\cdot L} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -g & 0 & 0 \\ 0 & 0 & g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & s\cdot C & 0 \\ 0 & 0 & 0 & 0 & 0 & R \end{bmatrix}$$

Very simular to the original circuit, just replace g by -g
The results of the adjoint circuits are:

$$\begin{bmatrix} Va2 \\ Va3 \\ Va4 \end{bmatrix} := \begin{bmatrix} \dfrac{s\cdot(L\cdot R\cdot g^2 + L\cdot g)}{(s\cdot(C + L\cdot g^2) + R\cdot g^2)} \\ \dfrac{s\cdot(R\cdot C - L\cdot g) + 1}{(s\cdot(C + L\cdot g^2) + R\cdot g^2)} \\ \dfrac{s\cdot(R\cdot C - L\cdot g) - R\cdot g}{(s\cdot(C + L\cdot g^2) + R\cdot g^2)} \end{bmatrix} \cdot(-i1a) \quad VNa := \begin{bmatrix} \dfrac{s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1}{(s\cdot(C + L\cdot g^2) + R\cdot g^2)} \\ \dfrac{s\cdot(L\cdot R\cdot g^2 + L\cdot g)}{(s\cdot(C + L\cdot g^2) + R\cdot g^2)} \\ \dfrac{s\cdot(R\cdot C - L\cdot g) + 1}{(s\cdot(C + L\cdot g^2) + R\cdot g^2)} \\ \dfrac{s\cdot(R\cdot C - L\cdot g) - R\cdot g}{(s\cdot(C + L\cdot g^2) + R\cdot g^2)} \\ \dfrac{1 + R\cdot g}{(s\cdot(C + L\cdot g^2) + R\cdot g^2)} \\ \dfrac{s\cdot(L\cdot R\cdot g^2 + R\cdot C) - R\cdot g}{(s\cdot(C + L\cdot g^2) + R\cdot g^2)} \end{bmatrix} \cdot(-i1a)$$

Since V1a=1

$$I1a := \frac{-(s\cdot(C + L\cdot g^2) + R\cdot g^2)}{s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1} \quad VNa := \begin{bmatrix} 1 \\ \dfrac{s\cdot(L\cdot R\cdot g^2 + L\cdot g)}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \\ \dfrac{s\cdot(R\cdot C - L\cdot g) + 1}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \\ \dfrac{s\cdot(R\cdot C - L\cdot g) - R\cdot g}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \\ \dfrac{1 + R\cdot g}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \\ \dfrac{s\cdot(L\cdot R\cdot g^2 + R\cdot C) - R\cdot g}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \end{bmatrix}$$

The solutions for both the original the adjoint circuit:

$$VN := \begin{bmatrix} 1 \\ \dfrac{s\cdot(L\cdot R\cdot g^2 - L\cdot g)}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \\ \dfrac{s\cdot(R\cdot C + L\cdot g) + 1}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \\ \dfrac{s\cdot(R\cdot C + L\cdot g) + R\cdot g}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \\ \dfrac{1 - R\cdot g}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \\ \dfrac{s\cdot(L\cdot R\cdot g^2 + R\cdot C) + R\cdot g}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \end{bmatrix} \qquad VNa := \begin{bmatrix} 1 \\ \dfrac{s\cdot(L\cdot R\cdot g^2 + L\cdot g)}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \\ \dfrac{s\cdot(R\cdot C - L\cdot g) + 1}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \\ \dfrac{s\cdot(R\cdot C - L\cdot g) - R\cdot g}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \\ \dfrac{1 + R\cdot g}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \\ \dfrac{s\cdot(L\cdot R\cdot g^2 + R\cdot C) - R\cdot g}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \end{bmatrix}$$

$$Yin(s) := \frac{s\cdot(C + L\cdot g^2) + R\cdot g^2}{s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1}$$

Calculate sensitivity with respect to L: $SY(C) := \frac{L}{Y}\cdot\frac{dY}{dL}$

$\frac{dY22}{dL} := \frac{-1}{s\cdot L^2}$ All the other terms are zero

$$V2 := \frac{s\cdot(L\cdot R\cdot g^2 - L\cdot g)}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \qquad V2a := \frac{s\cdot(L\cdot R\cdot g^2 + L\cdot g)}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)}$$

$$SY(L) := \frac{-s\cdot L\cdot g^2\cdot(R^2\cdot g^2 - 1)}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)\cdot(s\cdot(C + L\cdot g^2) + R\cdot g^2)}$$

Calculate sensitivity with respect to C: $SY(C) := \frac{C}{Y}\cdot\frac{dY}{dC}$

$\frac{dY55}{dC} := s$ All the other terms are zero

$$V5 := \frac{1 - R\cdot g}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)} \qquad V5a := \frac{1 + R\cdot g}{(s\cdot(L\cdot R\cdot g^2 + R\cdot C) + 1)}$$

$$SY(C) := \frac{-s \cdot C \cdot (R^2 \cdot g^2 - 1)}{(s \cdot (L \cdot R \cdot g^2 + R \cdot C) + 1) \cdot (s \cdot (C + L \cdot g^2) + R \cdot g^2)}$$

Calculate sensitivity with respect to R: $SY(C) := \frac{R}{Y} \cdot \frac{dY}{dR}$

$\frac{dY66}{dC} := \frac{-1}{R^2}$ All the other terms are zero

$$V6 := \frac{s \cdot (L \cdot R \cdot g^2 + R \cdot C) + R \cdot g}{(s \cdot (L \cdot R \cdot g^2 + R \cdot C) + 1)} \qquad V6a := \frac{s \cdot (L \cdot R \cdot g^2 + R \cdot C) - R \cdot g}{(s \cdot (L \cdot R \cdot g^2 + R \cdot C) + 1)}$$

$$SY(R) := \frac{-R \cdot (s^2 \cdot (L \cdot g^2 - C)^2 - g^2)}{(s \cdot (L \cdot R \cdot g^2 + R \cdot C) + 1) \cdot (s \cdot (C + L \cdot g^2) + R \cdot g^2)}$$

c) (5)

Calculate sensitivity directly from Yin(s)

$$Yin(s) := \frac{s \cdot (C + L \cdot g^2) + R \cdot g^2}{s \cdot (L \cdot R \cdot g^2 + R \cdot C) + 1}$$

$$SY(L) := \frac{-s \cdot L \cdot g^2 \cdot (R^2 \cdot g^2 - 1)}{(s \cdot (L \cdot R \cdot g^2 + R \cdot C) + 1) \cdot (s \cdot (C + L \cdot g^2) + R \cdot g^2)}$$

$$SY(C) := \frac{-s \cdot C \cdot (R^2 \cdot g^2 - 1)}{(s \cdot (L \cdot R \cdot g^2 + R \cdot C) + 1) \cdot (s \cdot (C + L \cdot g^2) + R \cdot g^2)}$$

$$SY(R) := \frac{-R \cdot (s^2 \cdot (L \cdot g^2 - C)^2 - g^2)}{(s \cdot (L \cdot R \cdot g^2 + R \cdot C) + 1) \cdot (s \cdot (C + L \cdot g^2) + R \cdot g^2)}$$