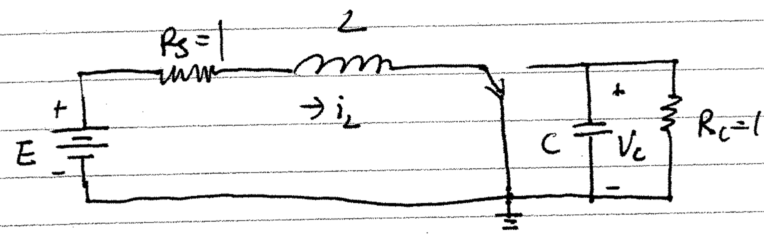
ENEE610 HW#1 09/13/2004

Problem 2.



for $t>0$,

$$\begin{cases} R_S \cdot i_L + L\dfrac{di_L}{dt} + V_c = E & ① \\ i_L = C\dfrac{dV_c}{dt} + \dfrac{V_c}{R_L} & ② \end{cases}$$

with initial conditions $t=0$

$$\begin{cases} V_c\big|_{t=0} = 0 & ③ \\ \dfrac{dV_c}{dt}\Big|_{t=0} = \dfrac{i_L}{C}\Big|_{t=0} = \dfrac{E}{R_S \cdot C} & ④ \end{cases}$$

From ① & ②

$$LC\frac{d^2V_c}{dt^2} + \left(R_S C + \frac{L}{R_L}\right)\frac{dV_c}{dt} + \left(1+\frac{R_S}{R_L}\right)V_c = E \quad ⑤$$

2nd Order inhomogeneos

Special solution for equation ⑤ $V_c = E/\left(1+\frac{R_S}{R_L}\right) = \frac{E}{2}$ ⑥

$$\Delta = \left(R_S C + \frac{L}{R_L}\right)^2 - 4\cdot LC\cdot\left(1+\frac{R_S}{R_L}\right) \quad ⑦$$

Critial damping $\Delta = 0$, for $R_S = R_L = 1$ $C^2 - 6LC + L^2 = 0$ ⑧

$C = (3 \pm 2\sqrt{2})L$ ⑨

general solution (Critial damping Case) for equation ⑤

$$V_c(t) = (A + Bt)e^{\lambda t} + \frac{E}{2} \quad ⑩$$

with $LC\lambda^2 + \left(R_S C + \frac{L}{R_L}\right)\lambda + \left(1+\frac{R_S}{R_L}\right) = 0$ ⑪

$C = (3\pm 2\sqrt{2})L \Rightarrow \lambda = -(2\mp\sqrt{2})\frac{1}{L}$ ⑫

Apply Boundary conditions ③ & ④

⑬

$$\begin{cases} A = -\dfrac{E}{2} \\ B = -A\lambda = -\dfrac{2\mp\sqrt{2}}{2}\dfrac{E}{L} \end{cases} \quad ⑭$$

∴ For $C = (3\pm 2\sqrt{2})L$

$$V_c(t) = \left(-\frac{E}{2} - \frac{2\mp\sqrt{2}}{2}\frac{E}{L}t\right)e^{-(2\mp\sqrt{2})\frac{t}{L}} + \frac{E}{2} \qquad t>0$$

$V_c(t) < E$ for $t>0$