

ENEE 610
 Homework Problems for Grading, Set 4 (100 points)
 Due at class M 10/25/04
 State, lossless synthesis

1. (25 points)

Determine if the following is true for a real-rational driving-point impedance $z(s)$:

A non-zero $z(s)$ is lossless and positive-real if and only if

- a) $z(1)$ is positive and
- b) All poles and zeros are simple and on the imaginary axis and
- c) Poles and zeros alternate

2. (25) points

a) Determine those constants a, b, c such that the following $z(s)$ is lossless positive-real.

$$z(s) = \frac{s^2(s^2+b)}{(s+a)(s^2+c)}$$

b) For one set of the values of a, b, c found in part a) give a partial fraction synthesis of $z(s)$, a synthesis by partial fractions of $y(s)=1/z(s)$ and one using the Richards' function with the gyrator-inductor 2-port coupling structures.

3. (50) points

A movable MEMS plate similar to, but somewhat different from, the one discussed in class has the describing equations:

$$m \frac{d^2x}{dt^2} = -b \frac{dx}{dt} - k(L_0 - x) + \frac{q^2}{2A\epsilon}$$

$$v = r \cdot i + v_c, \quad q = C v_c, \quad C = \frac{\epsilon A}{L_0 - x}, \quad i = \frac{dq}{dt}$$

Assume the plates are square and 100uM on a side and made of gold, the top movable one being 2uM thick. Assume $k=3.9 \times 10^{-9}$ Newtons/M, $b=1.1 \times 10^{-15}$ Newton-sec/M and the mass-density of gold $=19.3 \times 10^3$ Kgram/M³. Take $r=50$ Ohms and $\epsilon=8.85 \times 10^{-12}$ Farad/M.

- a) Assume a DC voltage V_{dc} of 2Volts, $v=V_{dc}=2$, to give L_0 for $x=0$. What is the value of L_0 and of the steady state charge $q=Q_{dc}$?
- b) Set up the state equations using $[x, dx/dt, q]^T$ as the state and implement with a PSpice schematic using capacitors, resistors, a voltage source and Gvalue components.
- c) Using Spice solve the state equations for the state vector when $v=V_{dc}+v_p$ where v_p is a 1millisec pulse of 1milli Volt amplitude.
- d) Linearize at the DC operating point and find the transfer function $X(s)/V(s)$.