ENEE 610 HW#3 10/11/04

Problem 2: state variable equations have the form of:

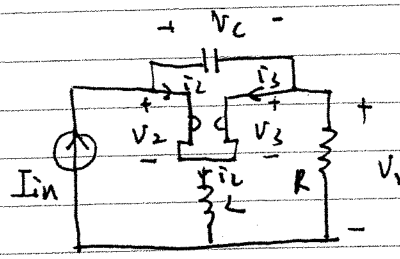
$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

x — state vector

u — input

y = output

For this problem $x = \begin{bmatrix} v_c \\ i_L \end{bmatrix}$, $u = I_{in}$, $Y = \begin{bmatrix} V_{in} \\ V_R \end{bmatrix}$



$$\left.\begin{array}{l} V_c = V_2 - V_3 \\ V_2 = -i_3/g \\ V_3 = i_2/g \end{array}\right\} \Rightarrow \left.\begin{array}{l} V_c = -(i_2+i_3)/g \\ i_2 + i_3 = i_L \end{array}\right\} \Rightarrow i_L = -gV_c \qquad (a)$$

$$\left.\begin{array}{l} i_c = I_{in} - i_2 = I_{in} - g\,v_3 \\ v_3 = v_R - V_2 = i_R \cdot R - V_2 = (I_{in} - i_L)R - V_L \end{array}\right\} \Rightarrow i_c - gV_L = gR\,i_L + (1-gR)I_{in} \qquad (b)$$

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$$\begin{cases} C\dot{V}_c - gL\dot{i}_L = gRi_L + (1-gR)I_{in} \\ g\dot{V}_c + \dot{i}_L = 0 \end{cases} \Rightarrow \begin{cases} (C+Lg^2)\dot{V}_c = -Rg^2V_c + (1-gR)I_{in} \\ (C+Lg^2)\dot{i}_L = -Rg^2 i_L - g(1-gR)I_{in} \end{cases} \qquad (c)$$

$$\begin{cases} V_R = Ri_R = (I_{in} - i_L)R = -Ri_L + RI_{in} \\ V_{in} = V_c + V_R = V_c - Ri_L + RI_{in} \end{cases} \qquad (d)$$

∴ state variable equations are: (one solution, not unique)

15'

$$\begin{bmatrix} C+Lg^2 & 0 \\ 0 & C+Lg^2 \end{bmatrix} \begin{bmatrix} \dot{V}_c \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -Rg^2 & 0 \\ 0 & -Rg^2 \end{bmatrix} \begin{bmatrix} V_c \\ i_L \end{bmatrix} + \begin{bmatrix} 1-gR \\ -g(1-gR) \end{bmatrix} I_{in}$$

5'

$$\begin{bmatrix} V_{in} \\ V_R \end{bmatrix} = \begin{bmatrix} 1 & -R \\ 0 & -R \end{bmatrix} \begin{bmatrix} V_c \\ i_L \end{bmatrix} + \begin{bmatrix} R \\ R \end{bmatrix} I_{in}$$

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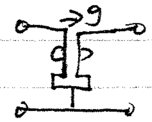
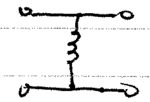
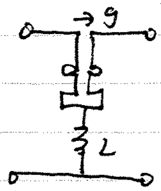
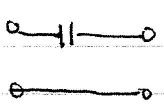
Problem 3

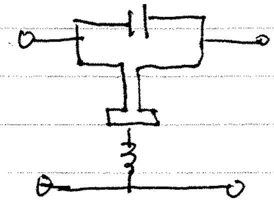
a)  $Y_g = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$ $Z_g = \begin{bmatrix} 0 & -\frac{1}{g} \\ \frac{1}{g} & 0 \end{bmatrix}$  $Z_L = \begin{bmatrix} sL & sL \\ sL & sL \end{bmatrix}$

10'

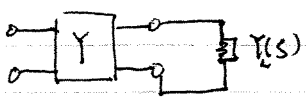


$Z_{gL} = Z_g + Z_L = \begin{bmatrix} sL & sL - \frac{1}{g} \\ sL + \frac{1}{g} & sL \end{bmatrix}$ $Y_{gL} = g^2 \begin{bmatrix} sL & -sL + \frac{1}{g} \\ -sL - \frac{1}{g} & sL \end{bmatrix}$

 $Y_C = \begin{bmatrix} sC & -sC \\ -sC & sC \end{bmatrix}$



$$Y = Y_C + Y_{gL} = \begin{bmatrix} s(C+Lg^2) & -s(C+Lg^2)+g \\ -s(C+Lg^2)-g & s(C+Lg^2) \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

5' b) 

$$\left.\begin{array}{l} i_1 = Y_{11}V_1 + Y_{12}V_2 \\ i_2 = Y_{21}V_1 + Y_{22}V_2 \end{array}\right\} \Rightarrow$$

also, $V_2 = -i_2 Y_L(s)$ $\Rightarrow V_2 = -\dfrac{Y_{21}}{Y_{22} + Y_L(s)} V_1$

$$i_1 = Y_{11}V_1 - \frac{Y_{12}Y_{21}}{Y_{22} + Y_L(s)} V_1$$

$$\therefore \; Y_{in} = \frac{i_1}{V_1} = \frac{\Delta Y + Y_{11} Y_L(s)}{Y_{22} + Y_L(s)} \quad \text{(a)} \quad \text{with } \Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21}$$

from (a) $Y_{in} = \dfrac{g^2 + s(C+Lg^2)Y_L(s)}{s(C+Lg^2) + Y_L(s)}$ (b)

5' c) $Y_L(s) = \dfrac{\Delta Y - Y_{22}Y_{in}}{Y_{in} - Y_{11}} = \dfrac{g^2 - s(C+Lg^2)Y_{in}(s)}{Y_{in}(s) - s(C+Lg^2)}$ (c)