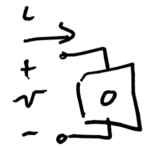
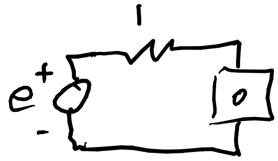
Passive 1-port $\Rightarrow$ nullator

$N_0 = \{[v, i] \mid v = i = 0\}$

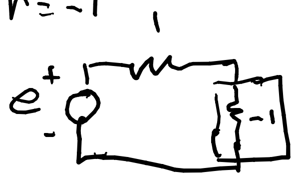
is passive





does not allow any $e \in L_2$ so can't define the scattering matrix

no $S(s)$ for $R = -1$



can't put any $e \in L_2$ on

Hurwitz polynomials:

$\dot{x} = Ax \qquad sX(s) - AX(s) = x(0)$

$(s1_k - A)X(s) = x(0)$

$X(s) = (s1_k - A)^{-1} x(0)$

$\det(s1_k - A) = P(s)$ occurs in the denominator of $X(s)$

$P(s)$ is Hurwitz if all zeros are in $\sigma \le 0$ & simple on $\sigma = 0$

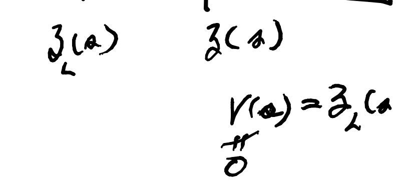
$P(s)$ is strictly Hurwitz " " " in $\sigma < 0$

a test: $P(s) = a_k s^k + a_{k-1} s^{k-1} + \dots + a_2 s^2 + a_1 s + a_0$

$= \sum_{i=0}^{k} a_i s^i$

look at reactance function, $\Rightarrow z(s)$ of a lossless passive 1-port

$z(s) = -z(-s) = \frac{Ev}{Od} = \frac{N(s)}{D(s)}$



$z_L(s) = \frac{1}{\frac{1}{R} + \frac{1}{z(s)}} = \frac{R\,z(s)}{R + z(s)} = \frac{R\,N/D}{R + N/D}$

$= \frac{RN}{N + RD}$

$\underset{0}{V(s)} = z_L(s) \cdot \underset{0}{I(s)} \Rightarrow$ nat freq at poles of $z_L(s) \Rightarrow$ same as zeros of $P(s) = N(s) + R\,D(s)$ these must be in $\sigma < 0$

we need $\frac{N}{D} = z(s) = \frac{Od}{Ev}$ to be a reactance function with no cancellations

Ex: $P(s) = s^4 + 3s^3 + 8s^2 + 2s + 5$ is this Hurwitz

$$Ev\,P(s) = s^4 + 8s^2 + 5$$

$$Od\,P(s) = 3s^3 + 2s$$

$$z(s) = \frac{s^4 + 8s^2 + 5}{(3s^2+2)s}$$ is this a reactance function

Make a partial fraction expansion

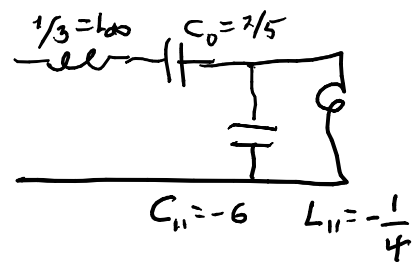
$$= \frac{1}{3}s + \frac{5/2}{s} + \frac{as+b}{3(s^2+2/3)} = \frac{\frac{1}{3}s^2 + 5/2}{s} + \frac{as+b}{3(s^2+2/3)}$$

$$\frac{as+b}{3(s^2+2/3)} = \frac{s^4+8s^2+5}{(3s^2+2)s} - \frac{\frac{1}{3}s^2+5/2}{s} = \frac{s^4+8s^2+5}{3(s^2+\frac{2}{3})s} - \frac{\frac{1}{3}(s^2+\frac{15}{2})(s^2+2/3)}{(s^2+\frac{2}{3})s}$$

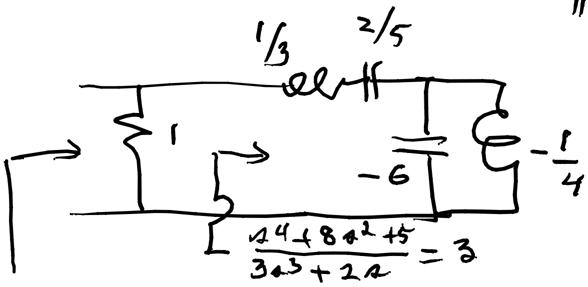
$$= \frac{\cancel{s^4} + 8s^2 \cancel{+5} - \cancel{s^4} - \frac{2}{3}s^2 - \frac{15}{2}s^2 \cancel{-5}}{3(s^2+2/3)s} = \frac{\left[8 - \frac{4-45}{6}\right]\cancel{s^2}}{3(s^2+2/3)\cancel{s}} = \frac{-\frac{1}{6}s}{3(s^2+2/3)}$$

is not a reactance function

$$z(s) = \frac{s^4+8s^2+5}{3(s^2+2/3)s} = \frac{1}{3}s + \frac{5/2}{s} + \frac{-\frac{1}{6}s}{s^2+2/3} \Rightarrow$$



$$y_{11} = \frac{s^2+2/3}{-1/6\,s}$$



$$\frac{s^4+8s^2+5}{3s^3+2s} = 3$$

$$z_L = \frac{\qquad}{s^4+3s^3+8s^2+2s+5}$$

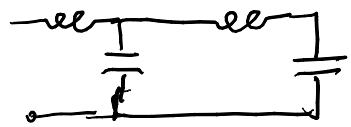
has a pole in RHP reasonable as have L_{11} & $C_{11} < 0$

Cauer (1st — extract poles at ∞ of a reactance function)

$$
\begin{array}{l}
3s^3+2s \;\big)\; s^4+8s^2+5 \quad \left(\tfrac{1}{3}s\right. \\
\qquad\qquad\;\; s^4+\tfrac{2}{3}s^2 \\
\qquad\qquad\qquad \tfrac{22}{3}s^2+5 \;\big)\; 3s^3+2s \quad \left(\tfrac{9}{22}s\right. \\
\qquad\qquad\qquad\qquad\qquad\quad 3s^3+\tfrac{45}{22}s \\
\qquad\qquad\qquad\qquad\qquad\qquad -\tfrac{1}{22}s \;\big)\; \tfrac{22}{3}s^2+5 \quad \left(-\tfrac{22^2}{3}s\right. \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \tfrac{22}{3}s^2 \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad 5 \;\big)\; -\tfrac{1}{22}s \quad \left(-\tfrac{1}{5\times 22}s\right.
\end{array}
$$

$$
Z(s) = \frac{1}{3}s + \cfrac{1}{\frac{9}{22}s + \cfrac{1}{-\frac{(22)^2}{3}s + \cfrac{1}{-\frac{1}{110}s}}}
$$

$L_1 = 1/3$ $\quad L_2 = -(22)^2/3$



$C_2 = -\frac{1}{110}$

$C_1 = \frac{9}{22}$

not passive

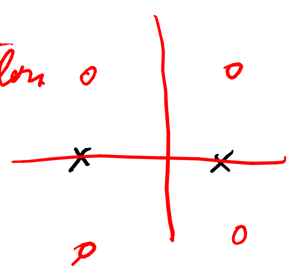
Note even factors can cancel

$$P(s) = (s^2+2)^2(s^4+s^2+5) + (s^2+2)^2 s(s^2+8)$$

$$= s^6 + \cdots \qquad \Rightarrow Z(s) = \frac{Ev}{od} = \frac{(s^2+2)(s^4+s^2+5)}{(s^2+2)s(s^2+8)}$$

$$= \frac{s^4+s^2+5}{s(s^2+8)}$$

Ev factors



If degree of $Z(s) = \frac{Ev}{od}$ < degree of $P(s)$ then there are even factors in the even and odd part of $P(s)$ & have to check them separately

- - - - - - -

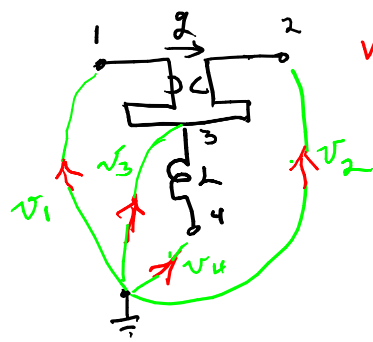
Ex: $\quad P(s) = s^5+3s^3+2s^2+2s+3$

$$Z(s) = \frac{2s^2+3}{s^5+3s^3+2s} \Rightarrow \frac{1}{3} = y \text{ has a 3rd order pole at } \infty$$

$$= \frac{2(s^2+3/2)}{s(s^4+3s^2+2)} = \frac{2(s^2+3/2)}{\underbrace{s(s^2+1)}(s^2+2)}$$

adjacent → poles & zeros don't alternate

indefinite Y



$$V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}, \quad \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = Y_{ind} V$$

$$g[V_2 - V_3] = I_1$$

$$-g[V_1 - V_3] = I_2$$

$$I_4 = \frac{1}{sL}[V_4 - V_3]$$

$$I_3 = \frac{1}{sL}(V_3 - V_4) + (-I_1) + (-I_2)$$

$$Y_{ind} = \begin{bmatrix} 0 & g & -g & 0 \\ -g & 0 & +g & 0 \\ g & -g & \frac{1}{sL} & -\frac{1}{sL} \\ 0 & 0 & -\frac{1}{sL} & \frac{1}{sL} \end{bmatrix}$$

all entries in a row sum to zero as do they for the rows

$$Y_{ind}\left(V + \begin{bmatrix} E \\ E \\ E \\ E \end{bmatrix}\right) = I = Y_{ind} V + \underbrace{\begin{bmatrix} \sum_{j=1}^{4} y_{1j} E \\ \vdots \\ \sum_{j=1}^{4} y_{4j} E \end{bmatrix}}_{0}$$

as holds for all E