See p. 77

$[v, i]$; v is an n-vector as is i

$N = \{[v, i] \mid v \,\&\, i \text{ allowed at the terminals}\}$

Passive: N is passive

$$\mathcal{E}(t) = \mathrm{Re}\int_{-\infty}^{t} v^{T*}(\tau)\, i(\tau)\, d\tau \geq 0 \text{ for all } t \text{ and all } [v, i] \in N$$

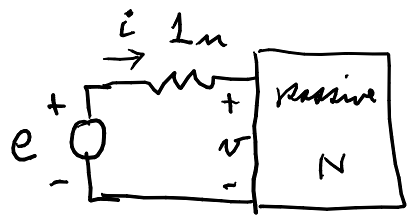
$\mathcal{E}(\infty)$ may be of interest, $*$ = complex conjugate when of interest

$x(t)$ is an L_2 function, scalar ($n=1$)

if $\int_{-\infty}^{\infty} x^*(\tau)\, x(\tau)\, d\tau$ exists (then is ≥ 0)

$$\|x\|^2_{L_2} = \langle x^*, x\rangle_{\text{vector}}$$

$$= \langle x^*, x\rangle_{L_2} \Rightarrow \langle x^*, y\rangle = \int_{-\infty}^{\infty} x^*(\tau)\, y(\tau)\, d\tau$$



assume $e \in L_2^n$ = n-vector square integrable function space

$$\|e\|^2_{L_2^n} = \int_{-\infty}^{\infty} e^T(\tau)\, e(\tau)\, d\tau$$

exists

$$= \int_{-\infty}^{\infty} (v(\tau) + i(\tau))^T (v(\tau) + i(\tau))\, d\tau = \|v\|^2_{L_2^n} + \|i\|^2_{L_2^n} + 2\langle v, i\rangle_{L_2^n}$$

$$\left(= 2\int_{-\infty}^{\infty} v^T(\tau)\, i(\tau)\, d\tau \geq 0 \text{ passive}\right.$$

if $e \in L_2^n$ then $v \,\&\, i \in L_2^n$ if passive

as $e = v + i = 2v^i$ know $v^i \in L_2^n$ given $e \in L_2^n$ then $v^i \in L_2^n$ and gives $v^r \in L_2^n$

$v - i = 2v^r$ then $v^r \in L_2^n$

Fourier transform

$$\mathcal{F}[\varphi(t)]_f = \int_{-\infty}^{\infty} \varphi(t) e^{-j2\pi f t}\, dt = \langle \varphi(t), e^{-j2\pi f t} \rangle$$

$$\mathcal{F}^{-1}[\Phi(f)]_t = \int_{-\infty}^{\infty} \Phi(f) e^{+j2\pi f t}\, df = \mathcal{F}^{*}[\Phi^{*}(f)]_t$$

$$\int_{-\infty}^{\infty} \varphi(t)\, \mathcal{F}[y(\eta)]_t\, dt = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \varphi(t)\, y(\eta)\, e^{-j2\pi t\eta}\, dt\, d\eta$$

$$= \int \mathcal{F}[\varphi]_\eta\, y(\eta)\, d\eta$$

Parseval's relation

Let $\varphi^{*}(t) = \mathcal{F}^{*}[\Phi(\eta)]_t$

$$\mathcal{F}^{-1}[\varphi(t)]_\eta = \mathcal{F}^{-1}[\mathcal{F}[\Phi(\eta)]_t]_\eta = \Phi(\eta)$$

$$\mathcal{F}^{*}[\varphi^{*}] =$$

$$\int_{-\infty}^{\infty} \mathcal{F}^{*}[x(\eta)]_f\, \mathcal{F}[y(\eta)]_f\, df$$

$$= \int_{-\infty}^{\infty} x^{*}(t)\, y(t)\, dt$$

here

$$\int_{-\infty}^{\infty} \mathcal{F}^{*}[\Phi(\eta)]_t\, \mathcal{F}[y(\eta)]_t\, dt$$

$$= \int_{-\infty}^{\infty} \mathcal{F}[\mathcal{F}^{*}[\Phi]]_\eta\, y(\eta)\, d\eta$$

$$= \int_{-\infty}^{\infty} \Phi^{*}(\eta)\, y(\eta)\, d\eta$$

$$\int_{-\infty}^{\infty} \Phi^{*}(t)\, y(t)\, dt = \int_{-\infty}^{\infty} \mathcal{F}^{*}[\Phi]\cdot \mathcal{F}[y]\, df$$

another form of Parseval's relation

shows that if $y \in L_2$ then $\mathcal{F}[y] \in L_2$

Now $V^{r}(j\omega) = S(j\omega)\, V^{i}(j\omega)$ and if $e \in L_2^n$ then $V^{i}(j2\pi f)$ & $V^{r}(j2\pi f)$ are L_2^n functions

∴ $S(j\omega)$ maps V^{i} into V^{r}

or maps finite energy functions into finite energy functions, so $S(j\omega)$ is a bounded map of L_2^n into L_2^n

We know $\|V^{r}(j\omega)\|_{L_2^n} = \|S(j\omega)\cdot V^{i}(j\omega)\|_{L_2^n}$ & $\|V^{i}(j\omega)\|_{L_2^n}$ exists & $\|V^{r}(j\omega)\|$ exists

$$\|S(j\omega)\| = \max \|V^{r}(j\omega)\| / \|V^{i}(j\omega)\|$$

we will have ≤ 1

$$\mathcal{E}(\infty) = \int_{-\infty}^{\infty} v^T(t)\, i(t)\, dt$$

$$v + i = 2v^i$$
$$v - i = 2v^n$$

$$= \int_{-\infty}^{\infty} (v^i + v^n)^T (v^i - v^n)\, dt$$

$$= \int_{-\infty}^{\infty} \left(v^{iT} v^i - v^{nT} v^n + [\underbrace{v^{nT} v^i - v^{iT} v^n}_{=0}] \right) dt$$

$$v^{nT} v^i = (v^{nT} v^i)^T = v^{iT} \cdot v^{nTT} = v^{iT} \cdot v^n$$

if passive

$$0 \leq \mathcal{E}(\infty) = \|v^i\|^2_{L_2^n} - \|v^n\|^2_{L_2^n}$$

$$= \|V^i(j\omega)\|^2_{L_{2n}} - \|V^n(j\omega)\|^2 = \|V^i(j\omega)\|^2 - \|S(j\omega) V^i(j\omega)\|^2$$

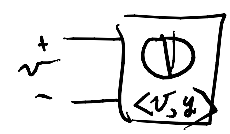
$$\overset{?}{\leq} \|V^i(j\omega)\|^2 \left(1 - \|S(j\omega)\|^2\right) \geq 0$$

$\Longrightarrow \quad 1 \geq \|S(j\omega)\|$ shows $S(s)$ is bounded on the $s = j\omega$

also will get $S(s)$ analytic in $\mathrm{Re}\, s > 0$

no poles on $s = j\omega$ from $1 \geq \|S(j\omega)\|$ (for all ω)

$$\langle \tilde{\Phi}^*, y \rangle = \langle \mathcal{F}^*[\Phi], \mathcal{F}[y] \rangle$$



$\tilde{\Phi}^* \in \mathcal{D}'$ = Distribution

$y \in \mathcal{D}$ = functions that are $\mathcal{D}'$ly differentiable & have compact support

$\mathcal{F}[y] = \mathcal{D}_{L_2}$

known if know Φ^* & y (& know $\mathcal{F}[y]$)