Lossless 1-port synthesis pp. 342–348

(call $z(s)$ a reactance function)

PR, $Z(s) = -Z^T(-s)$ $\qquad Z^T(-s) = Z_*$

i.e. $Z = -Z_*$

$\overset{*}{V}$ = complex conjugate

$\underset{*}{\wedge}$ = Hurwitz conjugate

4 classical syntheses

1. 1st Foster → partial fraction of Z
2. 2nd Foster → " " of $y = 1/Z$
3. 1st Cauer → continued fraction expansion about ∞
4. 2nd Cauer → " " " " 0

Ex: $z(s) = \dfrac{s(s^2+1)}{(s^2+1/2)(s^2+2)}$

1st Foster: $z(s) = \dfrac{k_1}{s+j\,1/\sqrt{2}} + \dfrac{k_1^*}{s-j\,1/\sqrt{2}} + \dfrac{k_2}{s+j\sqrt{2}} + \dfrac{k_2^*}{s-j\sqrt{2}}$

$$= \frac{(k_1+k_1^*)s + j\left(-\frac{k_1}{\sqrt{2}} + \frac{k_1^*}{\sqrt{2}}\right)}{s^2+1/2} + \frac{(k_2+k_2^*)s + j(-\sqrt{2}k_2 + \sqrt{2}k_2^*)}{s^2+2}$$

but j terms must disappear $k_1^* = k_1$, $k_2^* = k_2$

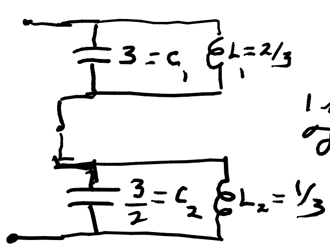
$$= \frac{2k_1 s}{s^2+1/2} + \frac{2k_2 s}{s^2+2}$$

need k_1 & k_2: $\dfrac{s^2+1/2}{s}\cdot z(s) = 2k_1 + 2k_2\dfrac{(s^2+1/2)}{(s^2+2)}$ set $s^2 = -1/2$

$$= \cancel{\frac{s^2+1/2}{s}} \times \frac{\cancel{s}(s^2+1)}{(\cancel{s^2+1/2})(s^2+2)} \Longrightarrow \frac{-1/2+1}{(-1/2+2)} = 2k_1 + 0$$

$\therefore\ 2k_1 = \dfrac{1/2}{3/2} = 1/3$; $\quad 2k_2 = \dfrac{\cancel{s}(s^2+1)}{(s^2+1/2)\cancel{(s^2+2)}} \times \cancel{\dfrac{s^2+2}{s}}\Bigg|_{s^2=-2} = \dfrac{-2+1}{-2+1/2} = \dfrac{-1}{-3/2} = \dfrac{2}{3}$

$$z(s) = \frac{s(s^2+1)}{(s^2+1/2)(s^2+2)} = \frac{\frac{1}{3}s}{s^2+1/2} + \frac{\frac{2}{3}s}{s^2+2} =$$



1st Foster

$$= \frac{1}{\frac{s^2+1/2}{1/3\,s}} \qquad = \frac{1}{\frac{3}{2}s + \frac{3}{s}} = Z_2$$

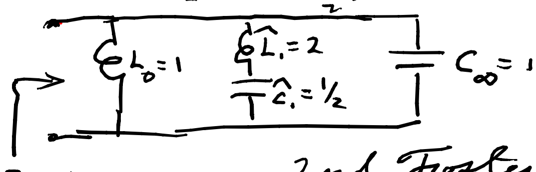
$Z_1 = \dfrac{1}{3s + \frac{3}{2s}} \qquad y_2 = \dfrac{3}{2}s + \dfrac{3}{s}$

$$y(s) = \frac{1}{z(s)} = \frac{(s^2+1/2)(s^2+2)}{s(s^2+1)} = \frac{k_0}{s} + \frac{2\hat{k}_1 s}{s^2+1} + k_\infty \cdot s = \frac{1}{s} + \frac{\frac{1}{2}s}{s^2+1} + s$$

$$k_0 = \frac{(0+1/2)(0+2)}{(0+1)} = 1$$

$$2\hat{k}_1 = \frac{(s^2+1/2)(s^2+2)}{s^2}\Bigg|_{s^2=-1} = \frac{(-1/2)(1)}{(-1)} = 1/2$$

$$\frac{z}{2} = 2s + \frac{1}{\frac{1}{2}s}$$



2nd Foster

1st Cauer – remove poles at ∞

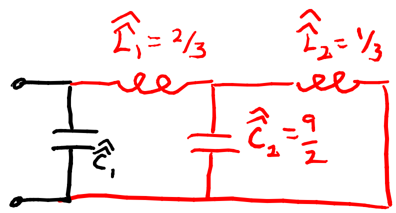
$$z(s) = \frac{s(s^2+1)}{(s^2+1/2)(s^2+2)} = \frac{1}{\frac{(s^2+1/2)(s^2+2)}{s(s^2+1)}} = \frac{1}{\frac{s^4+\frac{5}{2}s^2+1}{s^3+s}} = \frac{1}{s+\frac{\frac{3}{2}s^2+1}{s^3+s}}$$

$$= \frac{1}{s+\frac{1}{\frac{2}{3}s+\frac{\frac{1}{3}s}{\frac{3}{2}s^2+1}}}$$

$$\begin{array}{r} s \\ s^3+s \,\big)\, s^4+\frac{5}{2}s^2+1 \\ s^4+s^2 \\ \hline \frac{3}{2}s^2+1 \end{array} \qquad \begin{array}{r} 2s/3 \\ \frac{3}{2}s^2+1 \,\big)\, s^3+s \\ s^3+s\times 2/3 \\ \hline 0 \quad \frac{1}{3}s \end{array}$$

$$= \frac{1}{s+\frac{1}{\frac{2}{3}s+\frac{1}{\frac{\frac{3}{2}s^2+1}{1/3\,s}}}} = \frac{1}{s+\frac{1}{\frac{2}{3}s+\frac{1}{\frac{9}{2}s+\frac{1}{1/3\,s}}}}$$

$$z(s) = \frac{1}{s+\frac{1}{\frac{2}{3}s+\frac{1}{\frac{9}{2}s+\frac{1}{\frac{1}{3}s}}}}$$



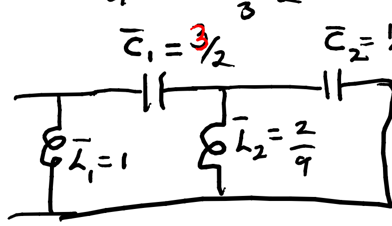
1st Cauer
(removed poles at ∞)
(good for low pass)

2nd Cauer – remove poles at $s=0$

$$z(s) = \frac{s(s^2+1)}{(s^2+1/2)(s^2+2)} = \frac{1}{\frac{s^4+\frac{5}{2}s^2+1}{s^3+s}} = \frac{1}{\frac{1}{s}+\frac{1}{\frac{1}{\frac{3}{2}s}+\frac{1}{\frac{1}{\frac{2}{9}s}+\frac{1}{\frac{1}{3}s}}}}$$

continued fraction expansion about $s=0$

$$\begin{array}{r} 1/s \\ s+s^3 \,\big)\, 1+\frac{5}{2}s^2+s^4 \\ 1+s^2 \\ \hline \frac{3}{2}s^2+s^4 \end{array} \qquad \begin{array}{r} \frac{1}{3/2\,s} \\ \frac{3}{2}s^2+s^4 \,\big)\, s+s^3 \\ s+\frac{2}{3}s^3 \\ \hline \frac{1}{3}s^3 \end{array} \qquad \begin{array}{r} \frac{9}{2s} \\ \frac{1}{3}s^3 \,\big)\, \frac{3}{2}s^2+s^4 \\ \frac{3}{2}s^2 \\ \hline s^4 \end{array} \qquad \begin{array}{r} \frac{1}{3s} \\ s^4 \,\big)\, \frac{1}{3}s^3 \\ \frac{1}{3}s^3 \end{array}$$

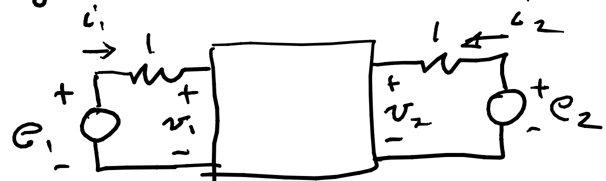


2nd Cauer

(good for high pass)

Here we have 4 circuits using the minimum number of reactive elements, the minimum number degree of $Z(s) = 4$, called canonical

$$2v^i = v + i = e$$
$$2v^r = v - i$$

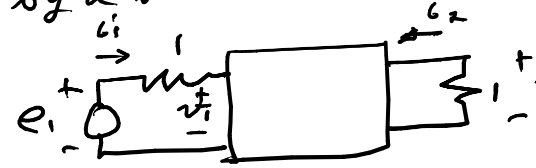


here when $e_2 = 0$

$$2v_2^r = v_2 - i_2 = 2v_2$$
$$v_2^r = v_2$$

if $e_2 = 0$ get "a load" on port 2 fed by a source on port 1

$$2v_1^i = e_1$$



$v_2 = -i_2$

$$V^r = S\,V^i$$

here $2v_2^i = v_2 + i_2 = 0$

$$\left.\begin{aligned} V_2^r &= S_{21} V_1^i + S_{22} V_2^i \\ &= S_{21} V_1^i \quad \text{as here } V_2^i = 0 \end{aligned}\right\} \quad V_2 = S_{21} \frac{E_1}{2}$$

$$\therefore \boxed{S_{21} = 2\frac{V_2}{E_1}}$$

$$S_{11} = \left.\frac{V_1^r}{V_1^i}\right|_{V_2^i = 0} = \text{reflection coefficient}$$

(when port 2 is terminated in $Z_0 = 1$)