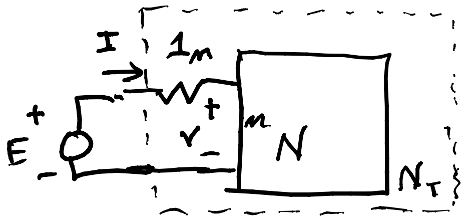
p. 178 = scattering matrix.

$$V^r = S(s)\, V^i$$

$$2V^i = V + I \qquad 2V = 2V^i + 2V^r$$
$$2V^r = V - I \qquad 2I = 2V^i - 2V^r$$

$$V = V^i + V^r$$
$$I = V^i - V^r$$




$$E = V + I = 2V^i$$
$$I = Y_T \cdot E$$
$$V = E - I = E - Y_T E = (1_m - Y_T)E$$

$$(1_m - Y_T)Y_T = Y_T - Y_T^2 = Y_T(1_m - Y_T)$$

$$\therefore \quad (1_m - Y_T)I = (1_m - Y_T)Y_T E = Y_T(1_m - Y_T)E = Y_T V$$

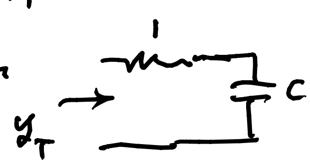
$$AV = BI\ ; \quad A = Y_T, \quad B = (1_m - Y_T)$$

$$A(V^i + V^r) = B(V^i - V^r) \implies (B + A)V^r = (B - A)V^i$$

$$V^r = (B + A)^{-1}(B - A)V^i = SV^i$$

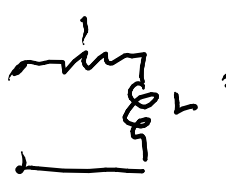
$$S = (B + A)^{-1}(B - A) = [(1_m - Y_T) + Y_T]^{-1}[(1_m - Y_T) - Y_T]$$
$$= 1_m - 2Y_T$$

$m = 1$, capacitor




$$\Rightarrow \quad y_T = \frac{1}{1 + 1/sC} = \frac{sC}{1 + sC}$$

$$s_C(s) = 1 - 2\left(\frac{sC}{1 + sC}\right) = \frac{1 - sC}{1 + sC}$$




$$\Rightarrow \quad y_T = \frac{1}{1 + sL}.$$

$$s_L(s) = 1 - 2\left(\frac{1}{1 + sL}\right) = \frac{-1 + sL}{1 + sL} \qquad \text{no poles on } s = j\omega$$

$AV = BI \qquad S = (B+A)^{-1}(B-A)$

$Y = B^{-1}A \ , \ Z = A^{-1}B$

$$S = (B[1_m + B^{-1}A])^{-1}(B[1_m - B^{-1}A]) = ([1_m + B^{-1}A]^{-1}\cancel{B^{-1}}\cdot\cancel{B}[1_m - B^{-1}A])$$

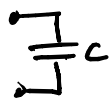
$$= (1_m + Y)^{-1}(1_m - Y) = (1_m - Y)(1_m + Y)^{-1}$$

$$(1_m + Y)(1_m - Y) = 1_m - Y^2 = (1_m - Y)(1_m + Y)$$

$$S = (Z - 1_m)(Z + 1_m)^{-1} = (Z + 1_m)^{-1}(Z - 1_m)$$

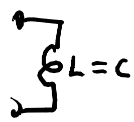
For the dual circuit $V \to I$ , $I \to V$ ; $I^d = V$, $V^d = I$

$$S^d = -S$$



$y_c = sC$

$S_c = (1 + sC)^{-1}(1 - sC)$



$z = sL$

$S_L = (sL - 1)(sL + 1)^{-1}$

$S_L^d = (sC - 1)(sC + 1)^{-1} = -S_c$

If $Z(s)$ is lossless then

$$Z(s) + Z^T(-s) = 0_m$$

but $\quad S(Z + 1_m) = Z - 1_m \Rightarrow Z - SZ = S + 1_m$

$(1_m - S)Z = S + 1_m$

$Z = (1_m - S)^{-1}(S + 1_m) = (S + 1_m)(1_m - S)^{-1}$

$$Z(s) + Z(-s)^T = (1_m - S)^{-1}(S + 1_m) + [(1_m - S(-s))^{-1}(1_m + S(-s))]^T$$

$$= (1_m - S(s))^{-1}(S(s) + 1_m) + (1_m + S^T(-s))(1_m - S^T(-s))^{-1} = 0_m$$

$$(S(s) + 1_m)(1_m - S^T(-s)) = -(1_m - S(s))(1_m + S^T(-s))$$

$$1_m + \cancel{S(s)} - \cancel{S^T(-s)} - S(s)S^T(-s) = -1_m + \cancel{S(s)} - \cancel{S^T(-s)} + S(s)S^T(-s)$$

$$2(1_m - S(s)S^T(-s)) = 0_m$$

∴ lossless condition on $S(s)$: $\quad S(s)S^T(-s) = 1_m$

∴ $S^{-1}(s) = S^T(-s) \qquad$ (also $S^T(-s)\cdot S(s) = 1_m$)

note $s_c(s) = \frac{1-sc}{1+sc}$ , $s_c(-s) = \frac{1+sc}{1-sc} \Rightarrow s_c(s)\, s_c(-s) = 1$

$$S = (Z+1_n)^{-1}(Z-1_n)$$

for a passive circuit $S$ is analytic in $\mathrm{Re}\, s > 0$

if (real) rational " " on $\mathrm{Re}\, s = 0$

$$1_n - S(s)S^{T*}(s) \geq 0_n \quad \text{in } \mathrm{Re}\, s \geq 0 \text{ if real rational}$$

$$2\,\mathrm{Re}\, V^{T*}I = V^{T*}I + V^{T}I^{*} = V^{T*}I + I^{T*}V \quad (s = j\omega)$$

$$= (v^i+v^r)^{T*}(v^i-v^r) + (v^i-v^r)^{T*}(v^i+v^r)$$

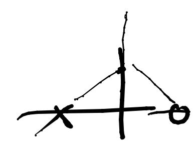
$$= [v^{iT*}v^i - \cancel{v^{iT*}v^r} + \cancel{v^{rT*}v^i} - v^{rT*}v^r] = 2(v^{iT*}[1_n - S^{T*}(s)S(s)]\Big|_{s=j\omega} v^i$$

$$+[v^{iT*}v^i + \cancel{v^{iT*}v^r} - \cancel{v^{rT*}v^i} - v^{rT*}v^r]$$

$$= 2V^{iT*}\underbrace{[1_n - S^{T*}(s)S(s)]\Big|_{s=j\omega}}_{\text{bounded for } s = j\omega \text{ if passive}} V^i \geq 0 \text{ if passive}$$

$$= 2[v^{iT*}v^i - v^{rT*}v^r] \quad (\text{sums of } |\cdot|^2 \text{ terms})$$

Ex: $S_c(s) = \frac{1-sc}{1+sc}$ passive if $c \geq 0$ , $s = -\frac{1}{c}$ a pole



$$1 - S_c^*(s)\, S_c(s) = 1 - \left(\frac{1-s^*c}{1+s^*c}\right)\left(\frac{1-sc}{1+sc}\right)$$

$$= 1 - \left(\frac{1 - c\,2\mathrm{Re}\, s + c^2|s|^2}{1 + c\cdot 2\mathrm{Re}\, s + c^2|s|^2}\right) \geq 0 \quad \text{checks passive condition}$$