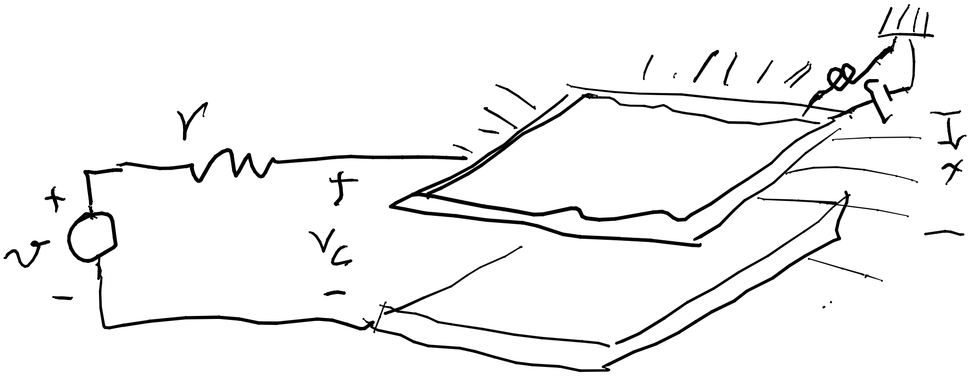
610_101104

modified 10/14/04



L_0 = plate separation when $Q=0$, i.e. $v = V_c = 0$

$$m\ddot{x} = -kx - b\dot{x} + \frac{Q^2}{\epsilon A}$$

$$C V_c = Q$$

$$C = \frac{\epsilon A}{L_0 - x}$$

$$v = ri + V_c = r\dot{Q} + \frac{Q}{C} = r\dot{Q} + \frac{(L_0 - x)\,x_3}{\epsilon A}$$

set for state $\underline{x} = [x, \dot{x}, Q]^T$ $\quad$ V^T = transpose

$x_1 = x$

$x_2 = \dot{x}$

$\dot{x}_2 = \ddot{x}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & r \end{bmatrix} \dot{\begin{bmatrix} x \\ \dot{x} \\ Q \end{bmatrix}} = \begin{bmatrix} 0 & 1 & 0 \\ -k & -b & 0 \\ 0 & 0 & -\frac{L_0}{\epsilon A} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ x_3^2/\epsilon A \\ \frac{x_1 x_3}{\epsilon A} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & 0 \\ 0 & 0 & -\frac{L_0}{\epsilon A r} \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ x_3^2/(\epsilon A m) \\ \frac{\cancel{x}_1 x_3}{\epsilon A r} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/r \end{bmatrix} u$$

$u = v$

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\dot{\underline{x}} = A\underline{x} + f(\underline{x}) + Bu$ $\quad$ state variable

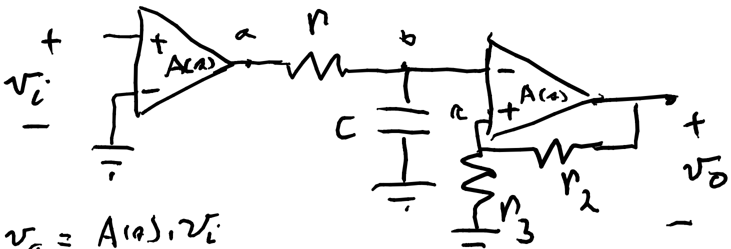
if desire V_c as the output; $V_c = \dfrac{Q}{C} = \dfrac{x_3 (L_0 - x_1)}{\epsilon A}$

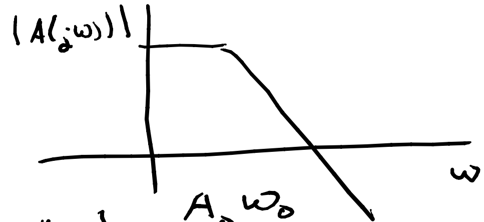
$y = V_c = g(\underline{x})$

To get $E\dot{x} = a(x) + Bu$, $y = Cx$, $\quad x_E = \begin{bmatrix} x \\ V_c \end{bmatrix}$

$$x_E = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}; \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \dot{x}_E = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & 0 & 0 \\ 0 & 0 & -\frac{L_0}{\epsilon A r} & 0 \\ 0 & 0 & +\frac{L_0}{\epsilon A} & -1 \end{bmatrix} x_E + \begin{bmatrix} 0 \\ x_3^2/(\epsilon A m) \\ x_1 x_3/(\epsilon A r) \\ -x_1 x_3/(\epsilon A) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/r \\ 0 \end{bmatrix} u$$

$$y = V_c = [0\ 0\ 0\ 1]\, x_E$$





$$A(s) = \frac{A_0\omega_0}{s+\omega_0}$$

1452 op amps
$A_0 \approx 10^5$
$\omega_0 \approx 10\,Hz$

$$v_a = A(s)\cdot v_i = \frac{A_0\omega_0}{s+\omega_0}\cdot v_i \qquad s\,v_a = -\omega_0 v_a + A_0\omega_0 v_i$$

$$v_b = \frac{1/sC}{r+\frac{1}{sC}}\cdot v_a = \frac{1}{rCs+1}\cdot v_a \Rightarrow s\,v_b = -\frac{1}{rC}v_b + \frac{1}{rC}v_a$$

$$v_o = A(s)(v_c - v_b) = \frac{A_0\omega_0}{s+\omega_0}\left(\frac{r_3}{r_2+r_3}v_o - v_b\right)$$

$$s\,v_o = (-\omega_0)\left(1 - \frac{A_0 r_3}{r_2+r_3}\right)v_o - A_0\omega_0 v_b$$

$$x = \begin{bmatrix} v_a \\ v_b \\ v_o \end{bmatrix} \Rightarrow \dot{x} = \begin{bmatrix} -\omega_0 & 0 & 0 \\ 1/rC & -1/rC & 0 \\ 0 & -A_0\omega_0 & \underbrace{\ }_{-\omega_0\left(1-\frac{A_0 r_3}{r_2+r_3}\right)} \end{bmatrix} x + \begin{bmatrix} A_0\omega_0 \\ 0 \\ 0 \end{bmatrix} v_i$$

$$v_o = y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x$$

$$T(s) = \frac{V_o}{V_i}(s) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s+\omega_0 & 0 & 0 \\ -1/rC & s+1/rC & 0 \\ 0 & -A_0\omega_0 & s+\omega_0\left(1-\frac{A_0 r_3}{r_2+r_3}\right) \end{bmatrix}^{-1} \begin{bmatrix} A_0\omega_0 \\ 0 \\ 0 \end{bmatrix}$$

$$= [1]\{\underbrace{(3,1)\text{ entry of inverse}}_{\frac{(1,3)\text{ cofactor}}{\text{determinant}}}\}[A_0\omega_0] = \frac{1\cdot(A_0\omega_0/rC)(A_0\omega_0)}{(s+\omega_0)(s+\frac{1}{rC})(s+\omega_0(1-\frac{A_0 r_3}{r_2+r_3}))}$$

$$(1,3)\text{ cofactor} = (-1)^{1+3}\cdot\begin{vmatrix} -1/rC & s+1/rC \\ 0 & -A_0\omega_0 \end{vmatrix} =$$

$$T(s) = \frac{k}{s^3+d_2 s^2+d_1 s+d_0} = \frac{(A_0\omega_0)^2/(rC)}{s^3+\left[(\omega_0+\frac{1}{rC})+\omega_0(1-A_0\hat{r})\right]s^2 + \left[\frac{\omega_0}{rC}+(\omega_0+\frac{1}{rC})(\omega_0(1-A_0\hat{r}))\right]s + \frac{\omega_0^2}{rC}(1-A_0\hat{r})}$$

with $\hat{r} = \frac{r_3}{r_2+r_3}$

for oscillations $v_o = v_i$

$V_o = T(s)\cdot V_i = V_i \quad (1 - T(s))V_i = 0$ if $V_i \neq 0$
have a natural response

at s such that $1 - T(s) = 0$
i.e. zeros of $s^3 + d_2 s^2 + d_1 s + (d_0 - k) = 0$

for sinusoidal oscillations, $s = j\omega \Rightarrow$ Ev & od both $= 0$

$$\left.\begin{array}{l} -\omega^3 + d_1\omega = 0 \\ -d_2\omega^2 + d_0 - k = 0 \end{array}\right\} \text{ need both of these}$$

$\Rightarrow \quad \omega_{osc}^2 = d_1, \quad d_1 d_2 = d_0 - k$

gives 2 eqs., 3 things free to satisfy them: ω_{osc}, RC, $\hat{n} < 1$

Scattering variables:

$2v^i = v + z_0 i$; assume z_0 real
$2v^r = v - z_0 i$ normalize to 1Ω

$2v^i = v + i$
$2v^r = v - i$

$v = v^i + v^r$
$i = v^i - v^r$