610_100604

State variable equations

A242 – linear time invariant

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du \quad \left(+ \overset{E_{dif}}{\dot{u}} \text{ terms } \ddot{u}, \dddot{u}\right)$$

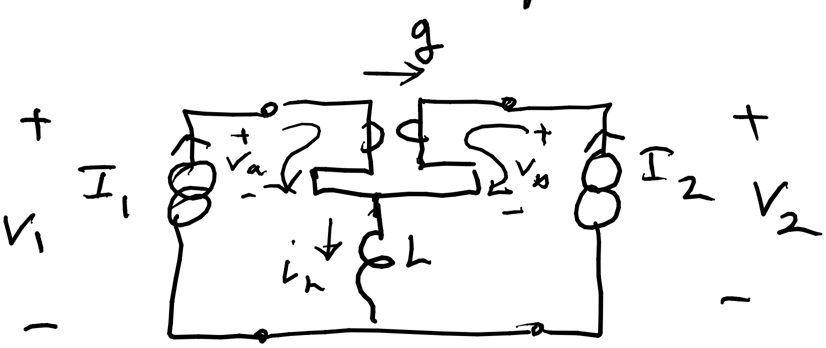
x = state vector

u = input vector

y = output vector

$v = sLi$ with $u = i$, $v = y$ needs $\dot{u}$

$$y = E_{dif}\dot{u}$$



$$u(t) = \begin{bmatrix} I_1(t) \\ I_2(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} V_1(t) \\ V_2(t) \end{bmatrix}$$

$$x = [i_L] = [x_1]$$

$$V_L = L\frac{di_L}{dt} =$$

$$V_1 = V_a + V_L \Rightarrow V_L = V_1 - V_a$$

$$V_2 = V_b + V_L \qquad\quad = V_2 - V_b$$

$$V_L = L\frac{dI_L}{dt}$$

$$s = \frac{d}{dt}$$

$$I_1 = gV_b, \quad I_2 = -gV_a$$

$$\Rightarrow V_L = V_1 + rI_2 \quad ; \quad i_L = I_1 + I_2$$

$$= V_2 - rI_1$$

$$L\dot{x} = V_1 + rI_2$$

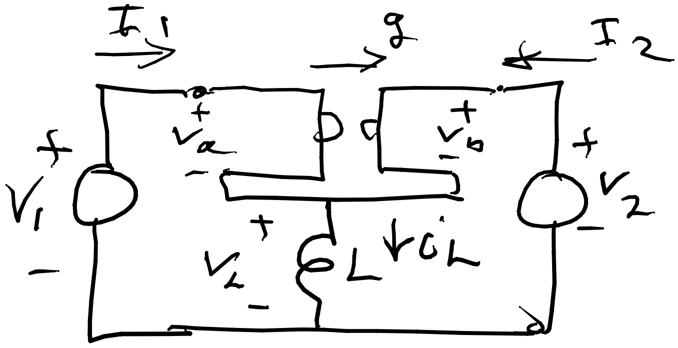
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} =$$

$$V = ZI$$

$$= \begin{bmatrix} sL & sL - r \\ sL + r & sL \end{bmatrix}$$

can not get in A, B, C, D form; need du/dt term

So take $u = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$, $y = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$



$v_1 = v_a + v_L$

$v_2 = v_b + v_L$

$i_L = I_1 + I_2$

$v_a = -r I_2$

$v_b = + r I_1$

$x = i_L \qquad L \dfrac{di_L}{dt} = v_L = v$

$v_1 = -r I_2 + v_L$

$v_2 = +r I_1 + v_L$

$I_1 + I_2 = i_L$

$I_1 - I_2 = g(v_1 + v_2) - 2g v_L$

$v_1 - v_2 = -r[I_2 + I_1] = -r\, i_L$

$v_1 + v_2 = r[I_1 - I_2] + 2v_L$

$2I_1 = i_L + g(v_1+v_2) - 2g v_L.$

$2I_2 = i_L - g(v_1+v_2) + 2g v_L$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} x + \frac{1}{2}\begin{bmatrix} g & g \\ -g & -g \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} -g \\ g \end{bmatrix} v_L$$

$$L\dot{x} = v_L$$

can take $v_L = x_2$, $x = x_1$, $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$y = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & g \\ \frac{1}{2} & g \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}g & \frac{1}{2}g \\ -\frac{1}{2}g & -\frac{1}{2}g \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$y = C\underline{x} + Du$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{L} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$E\dot{\underline{x}} = A\underline{x} + Bu$$

Equations of the form

$$E\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

are called semistate equations

$$sE\,X(s) = A\,X(s) + B\,U(s) \Rightarrow (sE - A)X(s) = BU(s)$$
$$Y(s) = C\,X(s) + D\,U(s)$$

$$Y(s) = C[sE - A]^{-1}B\,U(s) + D\,U(s)$$
$$= \left[C(sE-A)^{-1}B + D\right]U(s)$$

$$\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \Big[\underbrace{\underline{\underline{Y}}(s)}_{\text{admittance}}\Big] \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} \Rightarrow Y(s) = C[sE - A]^{-1}B + D$$

But here $sE-A$ is singular so something funny happens $Z(s) = \begin{bmatrix} sL & sL-r \\ sL+r & sL \end{bmatrix}$; $Y(s) = \frac{1}{r^2}\begin{bmatrix} sL & -(sL-r) \\ -(sL+r) & sL \end{bmatrix}$

still has a pole at infinity in $Y(s)$

Look at

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\dot{x} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}x + \begin{bmatrix} 0 \\ -1 \end{bmatrix}u$$

$$sE - A = \begin{bmatrix} -1 & s \\ 0 & -1 \end{bmatrix}; \quad (sE-A)^{-1} = \begin{bmatrix} -1 & -s \\ 0 & -1 \end{bmatrix}$$

$$y = \dot{u} \qquad y = [1 \quad 0]x$$

$$(sE-A)^{-1}B = \begin{bmatrix} -1 & -s \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} s \\ 1 \end{bmatrix}$$

$$C(sE-A)^{-1}B = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} s \\ 1 \end{bmatrix} = s$$

If I choose

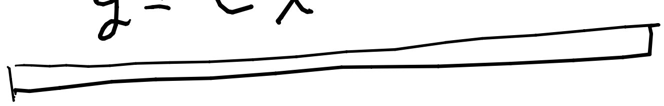
$$\left.\begin{aligned} &\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\dot{x} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ -1 \end{bmatrix} u \\ &y = \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{aligned}\right\} \quad \begin{aligned} E\dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

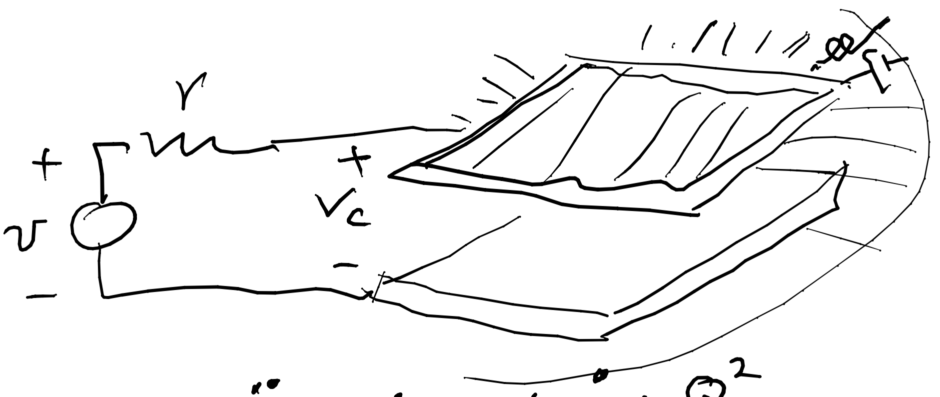
$$y(s) = T(s)\cdot u(s) \longrightarrow T(s) = s$$

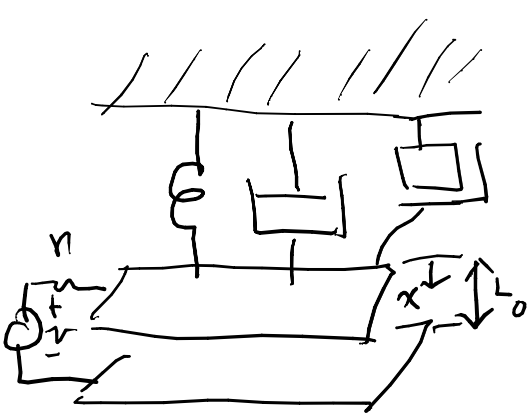
$$E\cdot E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \longleftarrow$$ a nilpotent matrix $E^i = \underline{\underline{0}}$, $E^{i-1} \neq \underline{\underline{0}}$

canonical form for "any" circuit is

$$E\dot{x} = \mathcal{A}(x,t) + Bu$$
$$y = Cx$$







$$m\ddot{x} = -kx - b\dot{x} + \frac{Q^2}{\epsilon A}$$

$$\vec{F} = Q\vec{E}\downarrow, \quad |E| = \frac{V_c}{L_0 - x}$$

$$Q = CV_c = \frac{\epsilon A}{L_0 - x}\cdot V_c$$

$$F = \frac{Q\,Q}{\epsilon A}$$

$$\underset{\sim}{x} = \begin{bmatrix} x \\ \dot{x} \\ Q \end{bmatrix}$$