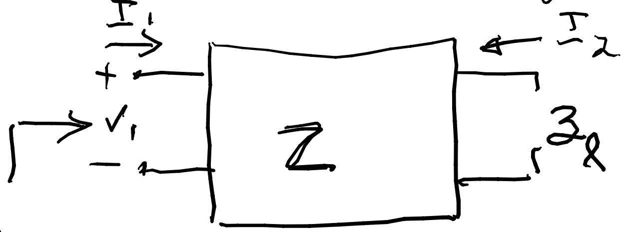
P. 361 = Richards' function



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix};$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

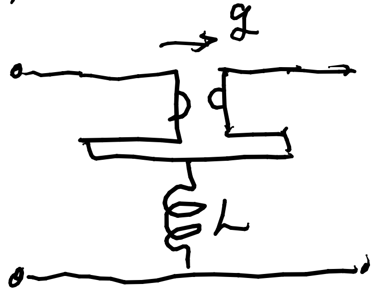
$$V_2 = Z_{21} I_1 + Z_{22} I_2 = -Z_\ell I_2 \Rightarrow I_2 = \frac{-Z_{21}}{Z_{22} + Z_\ell} I_1$$

$$V_1 = \left(Z_{11} - Z_{12} (Z_{22} + Z_\ell)^{-1} Z_{21} \right) I_1$$

$$Z_{in} = \frac{\Delta Z + Z_{11} Z_\ell}{Z_{22} + Z_\ell}$$ also find Z_ℓ vs Z_{in}

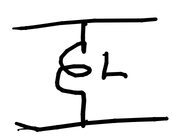
$$Z_{in} Z_{22} + Z_{in} Z_\ell = \Delta Z + Z_{11} Z_\ell \Rightarrow Z_\ell = \frac{Z_{in} Z_{22} - \Delta Z}{Z_{11} - Z_{in}}$$

Example



add Z's for

$$Y_{gy} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$$



$$Z_L = sL \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$Z_{gy} = Y_{gy}^{-1} = \frac{1}{g^2} \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1/g \\ 1/g & 0 \end{bmatrix} = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix}$$

$$Z = Z_{gy} + Z_L$$
$$= \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix} + \begin{bmatrix} sL & sL \\ sL & sL \end{bmatrix} = \begin{bmatrix} sL & sL - r \\ sL + r & sL \end{bmatrix};$$

$$\Delta Z = (sL)^2 - (sL - r)(sL + r) = r^2$$

$$Z_\ell = \frac{sL\, Z_{in}(s)\ r^2}{sL - Z_{in}(s)}$$

will want a zero in numerator and denominator at $s = k$
a factor $s - k/(s - k)$

$$z_\ell(k) = \frac{0}{0} = \frac{kL\,z_{in}(k) - r^2}{kL - z_{in}(k)}$$

$\therefore$ choose $L = z_{in}(k)/k$ forces denominator $= 0$

$$r^2 = k\,\frac{z_{in}(k)}{k}\,z_{in}(k)$$

$$\Rightarrow \quad r = \pm z_{in}(k)$$

$$\therefore \quad z_\ell(s) = \frac{\frac{s}{k}\cdot z_{in}(k)\,z_{in}(s) - z_{in}^2(k)}{\frac{s}{k}\cdot z_{in}(k) - z_{in}(s)}$$

checks $= \frac{0}{0}$ at $s = k$

$$= z_{in}(k)\left[\frac{\left(\frac{s}{k}\right)\left(\frac{z_{in}(s)}{z_{in}(k)}\right) - 1}{\left(\frac{s}{k}\right) - \left(\frac{z_{in}(s)}{z_{in}(k)}\right)}\right]$$

(this is $1/$Eq (8.6-1))

create a zero in numerator at $s = -k$

Note if use $z_{in}(k) = -z_{in}(-k)$ this will work

$$z_{in}(k) + z_{in}(-k) = 0$$

in s

$$2\,\mathrm{Ev}\,z_{in}(s) = z_{in}(s) + z_{in}(-s) = 0 \Rightarrow$$

desire k's which are zeros of $\mathrm{Ev}\,z_{in}$ $(k \neq 0, \infty)$

$$z_\ell(-k) = z_{in}(k)\,\frac{\left(\frac{-k}{k}\right)\left(\frac{-z_{in}(k)}{z_{in}(k)}\right) - 1}{\left(\frac{-k}{k}\right) - \frac{z_{in}(-k)}{z_{in}(k)}} = \frac{0}{0}$$

now degree of $z_\ell(s)$ = degree of $z_{in}(s) - 1$

Example: $z(s) = \dfrac{s+4}{s+1}$

$$z(s) + z(-s) = \frac{s+4}{s+1} + \frac{-s+4}{-s+1} = \frac{(s+4)(-s+1) + (-s+4)(s+1)}{-s^2+1}$$

$$= \frac{(-s^2 - \cancel{4s} + \cancel{s} + 4) + (-s^2 + 4 + \cancel{4s} - \cancel{s})}{-s^2+1}$$

$$= \frac{2(-s^2+4)}{-s^2+1} \Rightarrow 0 \; @ \; s = \pm 2, \; k = 2$$

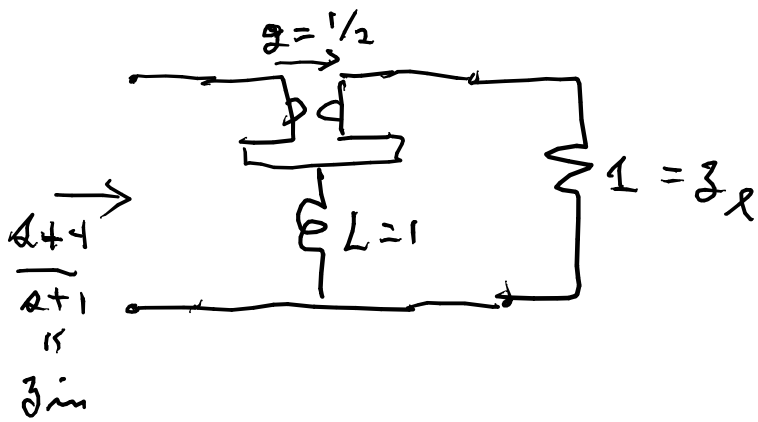
$$z_{in}(k) = z_{in}(2) = \frac{2+4}{2+1} = \frac{6}{3} = 2 \; ; \; L = \frac{z_{in}(k)}{k} = \frac{2}{2} = 1$$

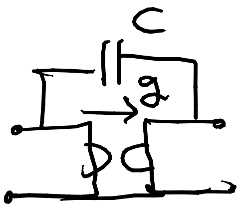
$$r = +z_{in}(k) = 2 ; \; g = 1/2$$

$$z_\ell(s) = z(2)\left[\frac{\frac{s}{2} \cdot \frac{z(s)}{z(2)} - 1}{\frac{s}{2} - \frac{z(s)}{z(2)}}\right] = 2\left[\frac{\frac{s}{2} \cdot \frac{1}{2}\left(\frac{s+4}{s+1}\right) - 1}{\frac{s}{2} - \frac{1}{2}\left(\frac{s+4}{s+1}\right)}\right]$$

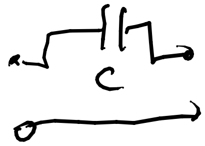
$$= 2\left[\frac{\frac{1}{4}\left[s(s+4) - 4(s+1)\right]}{\frac{1}{2}\left[s(s+1) - (s+4)\right]}\right] = \frac{\left[s^2 + 4s - 4s - 4\right]}{\left[s^2 + s - s - 4\right]}$$

$$= \frac{s^2-4}{s^2-4} = \frac{(s+2)(s-2)}{(s+2)(s-2)} = 1$$





$$Z = Y^{-1} \Rightarrow Y = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} + \begin{bmatrix} sC & -sC \\ -sC & sC \end{bmatrix}$$



$$= \begin{bmatrix} sC & g - sC \\ -g - sC & sC \end{bmatrix}$$

$$\Delta Y = (sC)^2 - (g - sC)(-g - sC) = g^2$$

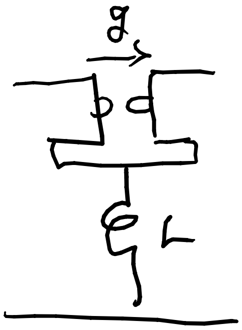
$$Z = \frac{1}{g^2} \begin{bmatrix} sC & -(g - sC) \\ g + sC & sC \end{bmatrix}$$

$$Z(s) = \begin{bmatrix} s\,C/g^2 & -\frac{1}{g} + s\frac{C}{g^2} \\ \frac{1}{g} + \frac{sC}{g^2} & s\frac{C}{g^2} \end{bmatrix} \equiv$$

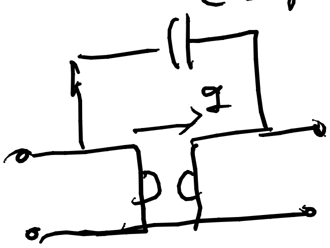
if set $C = g^2 L$

$$Z_{gyr\text{-}L} = \begin{bmatrix} sL & sL - r \\ r + sL & sL \end{bmatrix}$$

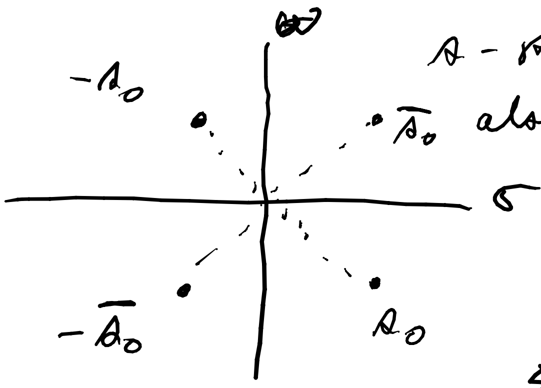
$C = g^2 L$



$\equiv$



∴ Using the Richards' function can get a cascade synthesis; if $Z_{in}(s)$ is positive real so is $Z_k(s)$ if $k > 0$ is chosen but note Even part zeros need not be real:



$Z(s) + Z(-s) = 0$

also true zeros of $Ev\, Z_{in}(s)$ are zeros of $Ev\, Z_{\ell}(s)$ except for $s^2 - k^2$ terms.

If $Z(s) = -Z(-s)$ can choose any real k

i.e. a reactance function (can be made with L's & C's)