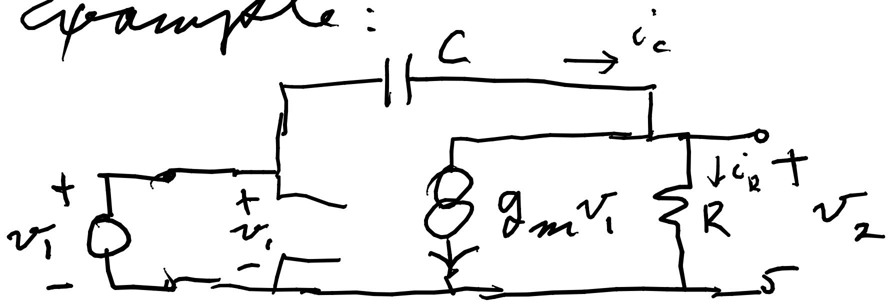
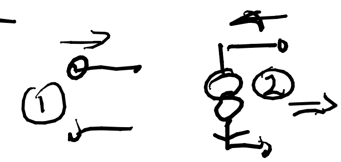
610_092704

Example:



$$T(s) = \frac{V_2}{V_1}(s)$$



$$\Rightarrow \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix}$$

find $S_C^{T(s)} = \dfrac{\partial T(s)/\partial C}{T(s)/C}$

if $\underline{\underline{V_2(s) = 1}}$ find $V_1(s) \Rightarrow$

$$\begin{aligned} 1 &= R \cdot i_R \\ &= R(i_C - g_m V_1) \\ &= R\left[Cs[V_1 - V_2] - g_m V_1\right] \\ &= RCs \cdot V_1 - RCs - Rg_m V_1 \end{aligned}$$

$$1 + RCs = [RCs - Rg_m]V_1$$

$$V_1 = \frac{1+RCs}{R(Cs - g_m)} \cdot 1 = V_2$$

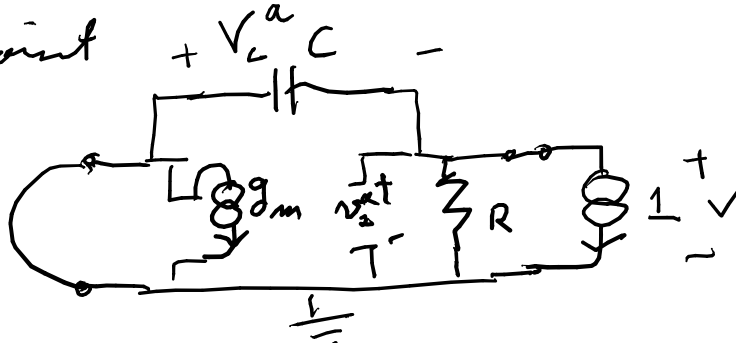
$$\frac{V_2}{V_1} = \frac{RCs - Rg_m}{RCs + 1} = T(s, g_m, R, C)$$

$$\frac{\partial T}{\partial C} = \frac{Rs}{(1+RCs)} - \frac{(RCs - Rg_m)\,Rs}{(RCs+1)^2} = \frac{Rs(RCs+1) - Rs(RCs - Rg_m)}{(1+RCs)^2}$$

$$= \frac{Rs(1 + Rg_m)}{(1+RCs)^2} \quad ; \quad S_C^T = \frac{\cancel{R}s(1+Rg_m)}{(1+RCs)^{\cancel{2}}\left(\dfrac{\cancel{R}Cs - \cancel{R}g_m}{\cancel{RCs+1}}\right)} \cdot \frac{1}{C}$$

$$\therefore \; S_C^T = \frac{Cs(1+Rg_m)}{(1+RCs)(Cs - g_m)}$$ is a function of s

adjoint



VCCS

$$Y = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix}$$

$$Y^a = \begin{bmatrix} 0 & g_m \\ 0 & 0 \end{bmatrix}$$

$$Y_{b\times b} = \begin{bmatrix} 0 & g_m & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & G & 0 \\ 0 & 0 & 0 & Cs \end{bmatrix} \Longrightarrow Y^a_{b\times b} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ g_m & 0 & 0 & 0 \\ 0 & 0 & G & 0 \\ 0 & 0 & 0 & Cs \end{bmatrix}$$

$$\frac{\partial Y_{b\times b}}{\partial C} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & s \end{bmatrix}$$ ∴ want V_c & V_c^a

then $$\frac{\partial V_2/V_1}{\partial C} = V_c^a \cdot s \cdot V_c$$

$V_1 = 1v$

$$-V_c = V_2 - V_1 = \frac{R(Cs - g_m)}{1+RCs} - 1 = \frac{-Rg_m - 1}{1+RCs}$$

$$1\text{ amp} = sCV_c^a - G\underbrace{V_2^a}_{= -V_2^a} = (sC+G)V_c^a$$

$$V_c^a = \frac{1}{sC+G} = \frac{R}{1+RCs}$$

$$\frac{\partial V_2/V_1}{\partial C} = V_c^a \cdot s \cdot V_c = \frac{s(+1)(1+Rg_m)\cdot R}{(1+RCs)^2}$$

checks direct calculation

$$S_a^{T(j\omega)} = \left|S_a^{T(j\omega)}\right| \cdot e^{j\angle S_a^{T(j\omega)}}$$

$$\left|S_a^T\right| = \frac{\left|\frac{\partial T(j\omega)}{\partial a}\right|}{\left|T(j\omega)/a\right|} \stackrel{?}{=} \frac{\frac{\partial |T(j\omega)|}{\partial a}}{|T(j\omega)|/a}$$ if a is real

$$= S_a^{|T(j\omega)|}$$

$$T(j\omega) = T_{Real} + j\,T_{Imag} = T_R + j\,T_I$$

$$|T(j\omega)| = \sqrt{T_R^2 + T_I^2}$$

$$\frac{\partial |T(j\omega)|}{\partial a} = \frac{1}{2}\frac{1}{\sqrt{T_R^2 + T_I^2}}\left(2\frac{\partial T_R}{\partial a} + 2\frac{\partial T_I}{\partial a}\right)$$

$$= \frac{1}{|T(j\omega)|}\cdot\left[\frac{\partial T_R}{\partial a} + \frac{\partial T_I}{\partial a}\right]$$

$$\left|\frac{\partial T(j\omega)}{\partial a}\right| = \left|\frac{\partial T_R}{\partial a} + j\frac{\partial T_I}{\partial a}\right| = \sqrt{\left(\frac{\partial T_R}{\partial a}\right)^2 + \left(\frac{\partial T_I}{\partial a}\right)^2}$$

not a nice relationship! But look at

$$S_a^{T(j\omega)} = S_a^{|T(j\omega)| e^{j\measuredangle T(j\omega)}}$$

$$= \frac{\partial |T(j\omega)|\cdot e^{j\measuredangle T(j\omega)} / \partial a}{\frac{|T(j\omega)|\cdot e^{j\angle jT(j\omega)}}{a}}$$

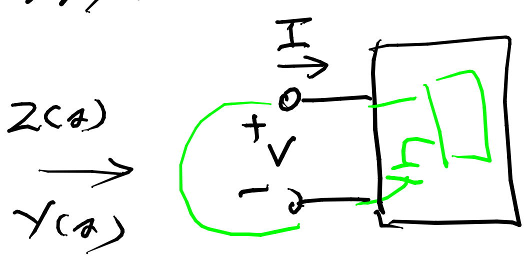
$$= \frac{\frac{\partial|T(j\omega)|}{\partial a}\cdot \cancel{e^{j\measuredangle T(j\omega)}}}{|T|\,\cancel{e^{j\measuredangle}}/a} + \frac{\cancel{|T(j\omega)|}\cdot \partial e^{j\measuredangle T(j\omega)}/\partial a}{\cancel{|T|}\,e^{j\measuredangle}/a}$$

$$e^{j\measuredangle T(j\omega)} \quad j\frac{\partial \measuredangle T(j\omega)}{\partial a}$$

$$= \underbrace{S_a^{|T(j\omega)|}}_{\substack{\text{real} \\ (a\ \text{real})}} + \underbrace{S_a^{\smile}}_{j\cdot\ \text{imaginary}}$$

$$\therefore \mathrm{Re}\left[S_a^{T(j\omega)}\right] = S_a^{|T(j\omega)|}$$ what normally would measure (note can replace $j\omega$ by s)

More on natural frequencies



$V = ZI$, $I = YV$

if open $I = 0$
if $V \neq 0 \Rightarrow Z = \infty$
$\Rightarrow$ a pole
$\therefore$ poles of Z (= zeros of Y) are open circuit natural frequencies

if short $V = 0$
$\Rightarrow Y$ has a pole $\equiv Z$ has a zero
$\therefore$ short circuit NF's are $Z = 0$, $Y = \infty$