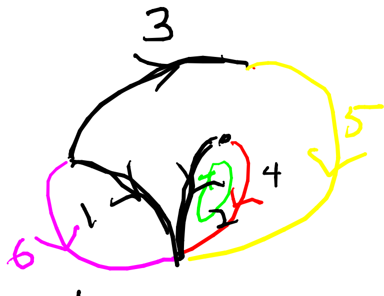
Problem 3.12, p. 124, gives a formula for the number of possible trees in a graph. There are lots of trees

$b = 6$
$n = 4$
$n-1 = 3 = t$
$\ell = b - t = 3$



tree

cotree

KVL

$$\begin{bmatrix}0\\0\\0\end{bmatrix} = \left[\begin{array}{ccc|ccc} 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix}_b = \mathcal{F} v_b$$

$$-B\mathcal{F}^T = -\begin{bmatrix} 1_4 & \\ & 0_2 \end{bmatrix} \begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & +1 & -1 \\ +1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & g & 0 & 0 & 0 & 0 \\ -g & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & sC & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \\ +1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & g & 0 \\ -g & 0 & 0 \\ 0 & 0 & sC \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = AC^T$$

$$\underbrace{\left[\begin{array}{ccc}0 & g & 0 \\ -g & 0 & 0 \\ 0 & 0 & sC \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ +1 & 0 & 0 \end{array}\right.}_{AC^T} \underbrace{\left|\begin{array}{ccc} 0 & +1 & -1 \\ +1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]}_{-B\mathcal{F}^T} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ v_s \end{bmatrix}}_{Ae - B_j}$$

4th eq $\rightarrow i_4 = 0$

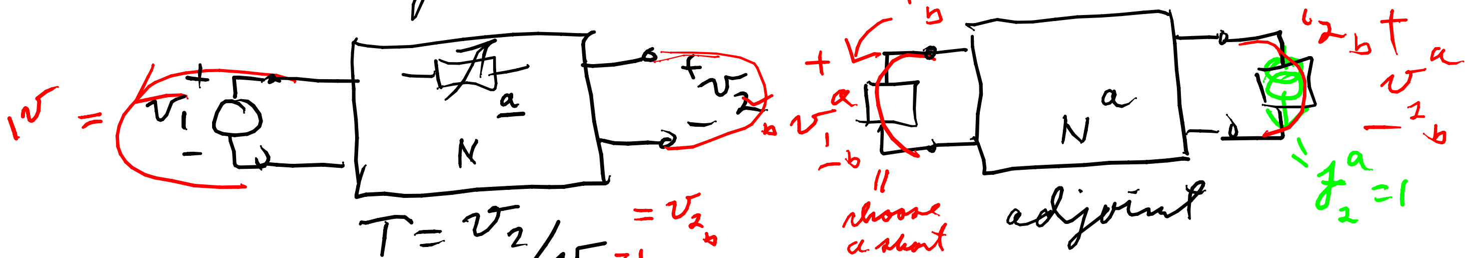
2nd eq $\rightarrow g v_1 = i_4 \rightarrow v_1 = 0$

last eq. $+v_1 = v_s \rightarrow v_s = 0$

Sensitivity: given $T(s,a)$; one parameter a which may change

$$S_a^{T(s)} = \frac{\partial T(s,a)/\partial a}{T(s,a)/a} = \frac{a}{T}\cdot\frac{\partial T}{\partial a}$$

Use the adjoint circuit



$T = v_2/v_1 = v_{2_b}$ ($v_1 = 1$)

N^a with the same graph as the original

$$v_b^T i_b^a - v_b^{aT} i_b = 0$$

assume N and N^a have 2x2 admittance matrices

$$\underbrace{v_{1_b} i_{1_b}^a}_{1=} + v_{2_b} \overset{=1}{i_{2_b}^a} + \underbrace{V_b^T I_b^a - V_b^{aT} I_b}_{\text{all internal branches}} - \overset{=0}{v_{1_b}^a} i_{1_b} - v_{2_b}^a \overset{=0}{i_{2_b}} = 0$$

assume the adjoint is fixed & does not change with the component which changes in N

$$i_b^a + v_{2_b} = V_b^{aT} Y V_b - V_b^T Y^a V_b^a$$

$$\frac{d}{d\underline{a}}:\ \overset{\shortparallel}{0} + \frac{\partial v_{2b}}{\partial \underline{a}} = V_b^{aT}\frac{dY}{d\underline{a}} V_b + \underbrace{V_b^{aT} Y \frac{dV_b}{d\underline{a}} - \frac{dV_b^T}{d\underline{a}} Y^a V_b^a}_{\text{choose } Y^a \text{ so these cancel}}$$

$$\Rightarrow \frac{\partial v_{2b}/1}{\partial \underline{a}} = V_b^{aT}\frac{dY}{d\underline{a}} V_b \qquad = V_b^{aT}\left[Y - Y^{aT}\right]\frac{dV_b}{d\underline{a}} \Rightarrow \text{choose } Y^a = Y^T$$

next time an example: