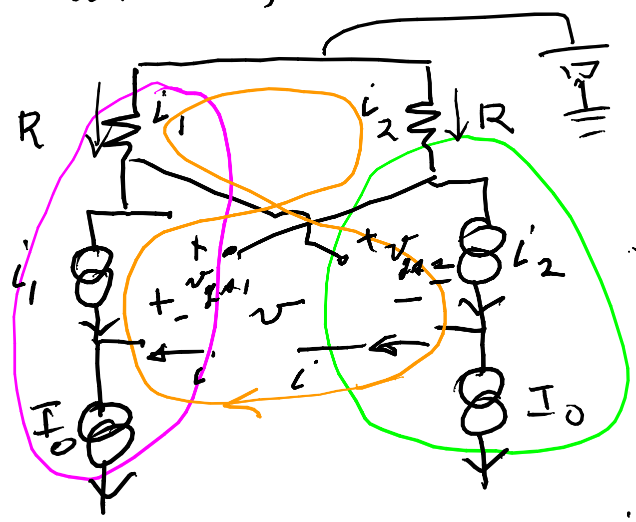
updated 09/22/04

610_092004

at DC, nonlinear



$$i_1 = I_0 - i = \beta(v_{gs_1} - V_{th})^2$$

$$i_2 = I_0 + i = \beta(v_{gs_2} - V_{th})^2$$

$$\Rightarrow \quad v_{gs_1} = V_{th} + \sqrt{\frac{I_0 - i}{\beta}} = V_{th} + \sqrt{\frac{I_0}{\beta}}\sqrt{1 - \frac{i}{I_0}}$$

$$v_{gs_2} = V_{th} + \sqrt{\frac{I_0 + i}{\beta}} = V_{th} + \sqrt{\frac{I_0}{\beta}}\sqrt{1 + \frac{i}{I_0}}$$

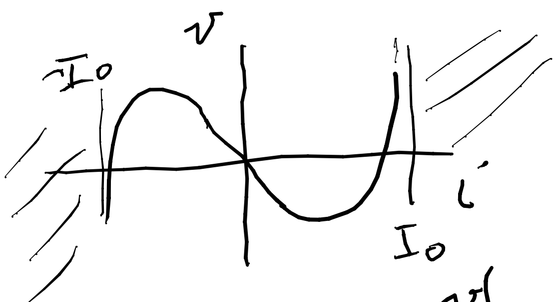
$$g_m = 2\sqrt{\beta I_0}$$

$$\text{KVL} \Rightarrow 0 = -v - v_{gs_1} - R i_2 + R i_1 + v_{gs_2}$$

$$\Rightarrow \quad v = \sqrt{\frac{I_0}{\beta}}\left(\sqrt{1+\frac{i}{I_0}} - \sqrt{1 - \frac{i}{I_0}}\right) + R(i_1 - i_2)$$

$$= R(-2i) + \frac{2}{2}\sqrt{\frac{I_0^2}{\beta I_0}}\left(\sqrt{1+\frac{i}{I_0}} - \sqrt{1 - i/I_0}\right)$$

$$= 2\left\{-Ri + \frac{I_0}{g_m}\left(\sqrt{1 + i/I_0} - \sqrt{1 - i/I_0}\right)\right.$$



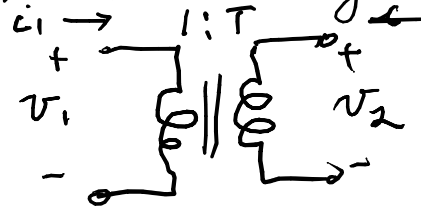
near $i = 0$; $|i/I_0| \ll \epsilon$ small

$$\sqrt{1+x} = 1 + \frac{x}{2} + \cdots, \quad |x| \text{ small}$$

$$v\Big|_{i\,\text{small}} = 2\left(-Ri + \frac{I_0}{g_m}\left[\cancel{1} + \frac{1}{2}\frac{i}{I_0} - \left(\cancel{1} - \frac{1}{2}\frac{i}{I_0}\right)\right]\right)$$

$$= 2\left(-Ri + \frac{I_0}{g_m}\left[\frac{i}{I_0}\right]\right) = 2i\left(-R + \frac{1}{g_m}\right)$$

Ideal transformer



$$v_2 = T v_1 \Rightarrow T v_1 - v_2 = 0$$

$$P_{in} = 0 = v_1 i_1 + v_2 i_2$$
$$= v_1 i_1 + T v_1 i_2$$
$$= v_1 (i_1 + T i_2)$$

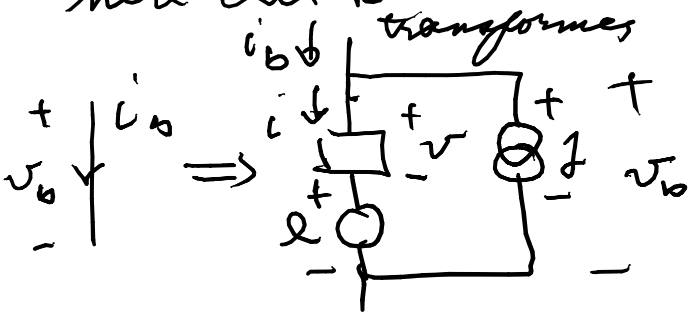
if for any v_1 then $i_1 + T i_2 = 0 \Rightarrow i_1 = -T i_2$

$i \neq Y v$; $\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = i$, $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = v$

$$\begin{bmatrix} T & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & T \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \Rightarrow \boxed{Av = Bi}$$

$$i = Yv \Rightarrow B^{-1}Bi = B^{-1}Av = i;\ Y = B^{-1}A$$

here $\det B = 0$ so no B^{-1} exists



$v_b = v + \ell$ ℓ = "ell"

$i_b = i + j$

$0 = \mathcal{C} i_b$ $v_b = \mathcal{C}^T v_t$

$0 = \mathcal{J} v_b$ $i_b = \mathcal{J}^T i_\ell$

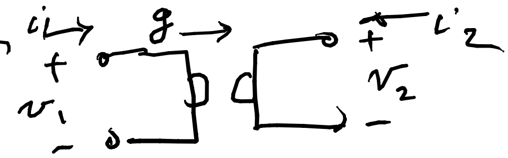
$$Av = A(v_b - \ell)$$
$$Bi = B(i_b - j)$$
$$\Rightarrow A(\mathcal{C}^T v_t - \ell) = B(\mathcal{J}^T i_\ell - j)$$

$$A\mathcal{C}^T v_t - B\mathcal{J}^T i_\ell = A\ell - Bj$$

$$b\left\{ \underbrace{\begin{bmatrix} \underbrace{A\mathcal{C}^T}_{t} & \underbrace{-B\mathcal{J}^T}_{\ell} \end{bmatrix}}_{b = t + \ell} \right. \begin{bmatrix} v_t \\ i_\ell \end{bmatrix} = A\ell - Bj = \text{source terms}$$

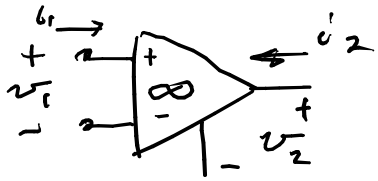
gives b equations in b unknowns (v_t & i_ℓ)

Example: Use a gyrator

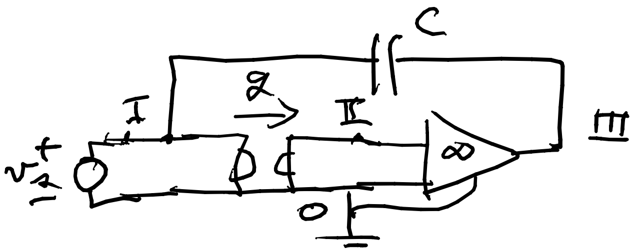


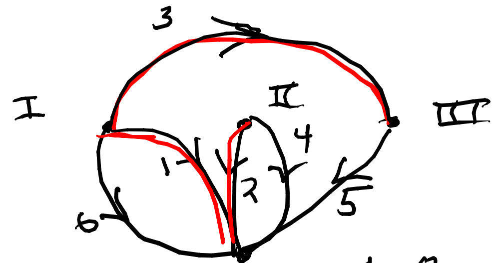
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

op-amp



$$v_1 = 0, \quad i_1 = 0 \Rightarrow \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$





$$\begin{bmatrix} 0 & g & 0 & 0 & 0 & 0 \\ -g & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & AC & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} \begin{matrix} 0 \\ j \end{matrix}$$

$j = \underline{0}$

$\ell = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ v_s \end{bmatrix}$

KVL:
$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & +1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{array}\right] \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \underline{0} = C\, i_b$$

For homework, problem #1, use branches <u>3, 4, 5, 6</u> as the tree