corrected 610_091504c

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ g_m & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g_m & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix} = e\underset{6\times 6}{Y}\, e^T$$

$$= \begin{bmatrix} g_m & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g_m & 0 & 0 & 0 & 0 & 0 \\ -g_m & g_m & 0 & 0 & G & 0 & 0 \\ g_m & -g_m & 0 & 0 & 0 & G & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} g_m & 0 & 0 & 0 \\ 0 & g_m & 0 & 0 \\ -g_m & g_m & G & 0 \\ g_m & -g_m & 0 & G \end{bmatrix}$$

$$-e\,j = -\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ i \end{bmatrix} = \begin{bmatrix} -i \\ i \\ i \\ -i \end{bmatrix}$$

$\therefore$ $-e\,j = e\,Y_{6\times 6}\,e^T v_t$ is to solve for v_t

$$\begin{bmatrix} -i \\ i \\ i \\ -i \end{bmatrix} = \begin{bmatrix} g_m & 0 & 0 & 0 \\ 0 & g_m & 0 & 0 \\ -g_m & g_m & G & 0 \\ g_m & -g_m & 0 & G \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_5 \\ v_6 \end{bmatrix}$$

add 1st row to 3rd
then subtract 2nd from 3rd, then add 2nd to fourth
and finally subtract 1st row from 4th

This gives
$$\begin{bmatrix} -i \\ i \\ -i \\ i \end{bmatrix} = \begin{bmatrix} g_m & 0 & 0 & 0 \\ 0 & g_m & 0 & 0 \\ 0 & 0 & G & 0 \\ 0 & 0 & 0 & G \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_5 \\ v_6 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_2 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} -i/g_m \\ i/g_m \\ -i/G \\ i/G \end{bmatrix}$$

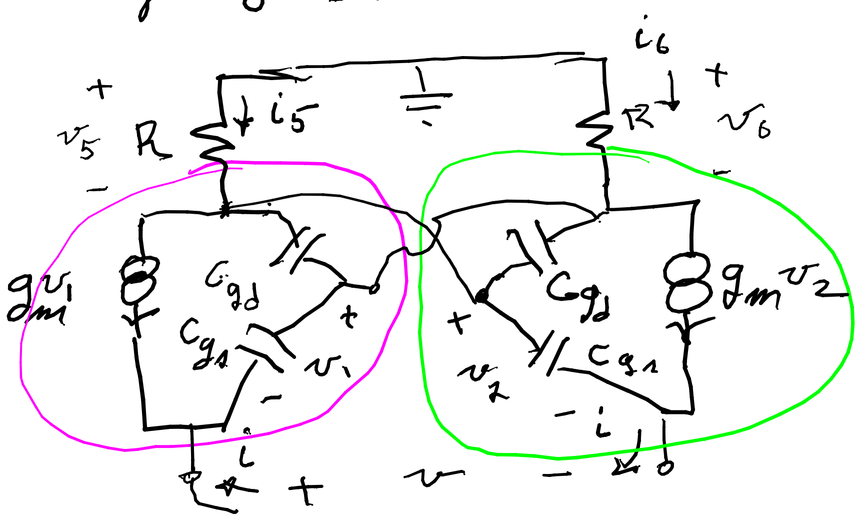
From the tie set matrix $0 = -v_1 + v_2 + v_5 - v_6 + v_7$

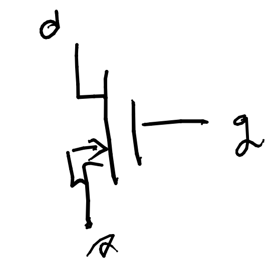
or $v = -v_7 = -v_1 + v_2 + v_5 - v_6 = \frac{i}{g_m} + \frac{i}{g_m} - \frac{i}{G} - \frac{i}{G} = \left(\frac{2}{g_m} - \frac{2}{G}\right) i$

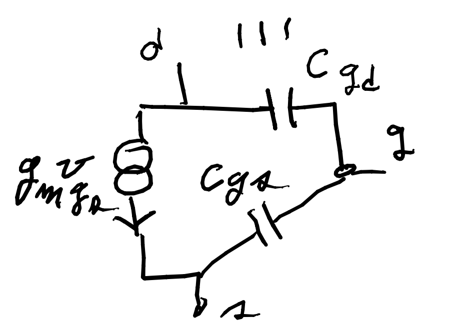
or $\frac{v}{i} = z_{in} = 2\left(\frac{1}{g_m} - \frac{1}{G}\right)$ note: $z_{in} < 0$ if $R > 1/g_m$

$= 2\left(\frac{1}{g_m} - R\right)$

To get $Z(s)$, insert transistor capacitors (small signal)







1) $i = -g_m v_1 - sC_{gs} v_1 = -(g_m + sC_{gs}) v_1$

$= g_m v_2 + sC_{gs} v_2 = (g_m + sC_{gs}) v_2 \Rightarrow v_2 = -v_1$

2) $0 = -v - v_1 - v_6 + v_5 + v_2 \Rightarrow v = -2v_1 + v_5 - v_6$

$i_5 = +(1/R) v_5 = g_m v_1 + sC_{gd}(v_2 \overset{-v}{-} v_1) + sC_{gd}(v_2 \overset{-v}{-} v_1) + sC_{gs} v_2$

$= (g_m - 4sC_{gd} - sC_{gs}) v_1 - 2sC_{gd} v$

3) $i_6 = (1/R) v_6 = (g_m v_2 + sC_{gd}(v_1 \overset{+v}{-} v_2) + sC_{gd}(v_1 \overset{+v}{-} v_2) + sC_{gs} v_1) + 2sC_{gd} v$

$= (-g_m + 4sC_{gd} + sC_{gs}) v_1 \overset{+2sC_{gd}v}{\wedge} = -i_5 \Rightarrow v_5 = -v_6$

2') $v = -2v_1 + v_5 - v_6 = -2v_1 - 2v_6 = -2(v_1 + v_6)$

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from 3)

4) $i_6 = -i_5, \quad v_6 = -v_5, \quad v_6 = [-g_m R + sR(4C_{gd} + C_{gs})]v_1 + 2RsC_{gd}v$

from 2') & 1)

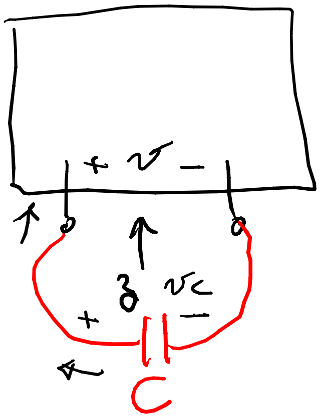
5) $v = -2[1 - g_m R + sR(4C_{gd} + C_{gs})]\left[\frac{-i}{g_m + sC_{gs}}\right] - 4sRC_{gd}v$

which is

6) $$\frac{v}{i} = Z(s) = \frac{2[(1-g_m R) + sR(4C_{gd}+C_{gs})]}{(g_m + sC_{gs})(1 + s4RC_{gd})}$$

two real LHP poles, at $s_1 = -\frac{g_m}{C_{gs}}$ & $s_2 = -\frac{1}{4RC_{gd}}$

one zero at ∞ and one real zero, at $s_3 = \frac{-(1-g_m R)}{R(4C_{gd}+C_{gs})}$ [may be in RHP if $g_m R > 1$]



$$v - v_c = 0 \Rightarrow \left(Z(s) + \frac{1}{sC}\right) \cdot i = 0$$

$$\underset{Z(s)\cdot i}{\overset{\parallel}{v}} \quad \underset{-\frac{1}{sC}\cdot i}{\overset{\parallel}{v_c}}$$

look for zeros of $\left(Z(s) + \frac{1}{sC}\right)$; nonzero i can then flow

i.e. zeros of $Z(s) + \frac{1}{sC}$ are the natural frequencies

from 6)

$$\frac{2[(1-g_m R) + sR(4C_{gd}+C_{gs})]}{(g_m + sC_{gs})(1 + s4RC_{gd})} + \frac{1}{sC} = \frac{N(s)}{D(s)}$$

zeros are zeros of

7) $N(s) = 2sC[(1-g_m R) + sR(4C_{gd}+C_{gs})] + [g_m + s(C_{gs} + 4g_m RC_{gd}) + s^2 4RC_{gs}C_{gd}]$

$= s^2[2RC(4C_{gd}+C_{gs}) + 4RC_{gs}C_{gd}] + s[2C(1-g_m R) + (C_{gs} + 4g_m RC_{gd})] + g_m$

For $N(s) = a[s^2 + \omega_0^2]$, oscillations, we want the coefficient of $s \Rightarrow 0$:

8) $\therefore \; 2C(1-g_m R) + (C_{gs} + 4g_m RC_{gd}) = 0$

which is

9) $$g_m R = \frac{-2C - C_{gs}}{-2C + 4C_{gd}} \Rightarrow R = \frac{1}{g_m} \cdot \frac{2C + C_{gs}}{2C - 4C_{gd}} > \frac{1}{g_m}$$

as $R > 1/g_m$ is the condition for $Z(0) < 0$ we see it is the negative resistance which allows small signal oscillations. Under the condition 9) the oscillation frequency is, from 7)

10) $$\omega_0 = 2\pi f_0 = \sqrt{\frac{g_m}{2R\left[C(4C_{gd} + C_{gs}) + 2C_{gs}C_{gd}\right]}}$$

$$= g_m \sqrt{\frac{2C - 4C_{gd}}{2(2C + C_{gs})\left[C(4C_{gd} + C_{gs}) + 2C_{gs}C_{gd}\right]}}$$

which, as, $g_m = 2\sqrt{\beta I_0}$, can be adjusted by I_0 (if also R is, by 9))