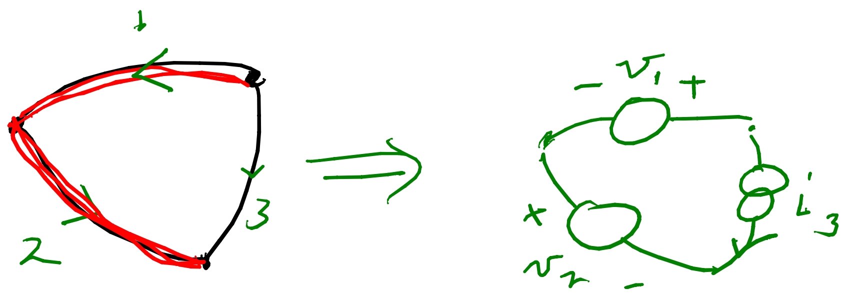
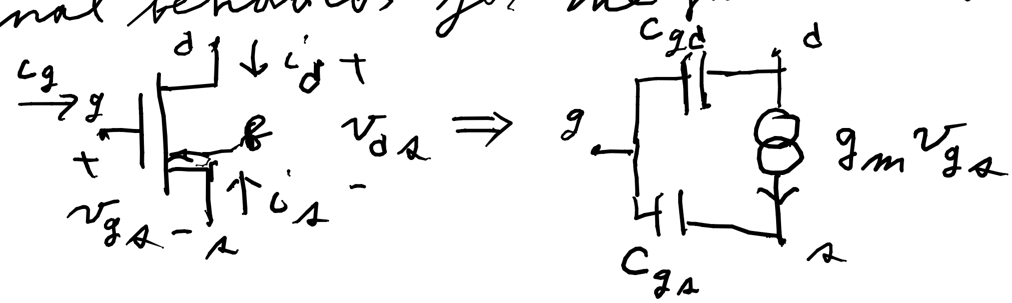


small signal behavior for the previous circuit



For small frequencies – ignore the capacitors assume transistors in saturation, ignore Early effect $\Rightarrow \lambda = 0$

$$i_D = \beta(v_{GS} - V_{th})^2 = I_D + \left.\frac{\partial i_D}{\partial v_{GS}}\right| (v_{GS} - V_{GS}) + \cdots$$

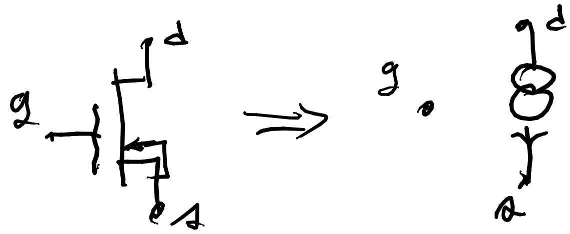
$$i_d = i_D - I_D = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{\text{bias}} v_{gs} + (\text{higher order at bias}) \longrightarrow \text{ignore for } |v_{gs}| \text{ small}$$

$$= g_m v_{gs}$$

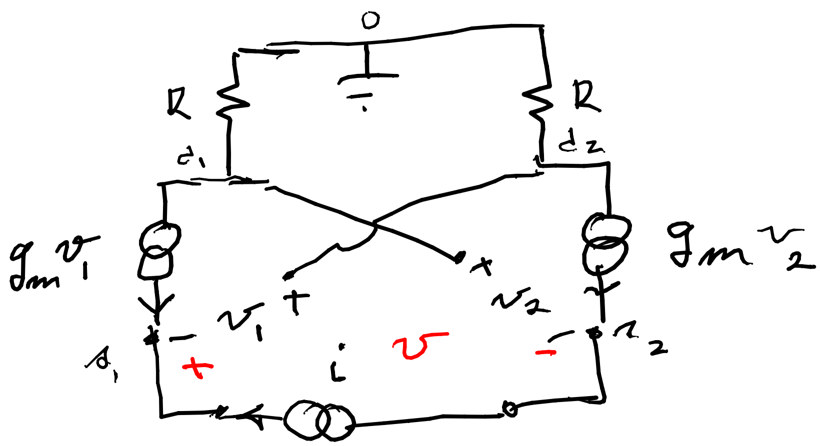
g_m is found at $I_D = I_O$

$$g_m = 2\beta(v_{GS} - V_{th})\Big|_{v_{GS} = V_{GS}} = \frac{2 I_O}{(V_{GS} - V_{th})}, \quad V_{GS} - V_{th} = \sqrt{\frac{I_O}{\beta}}$$

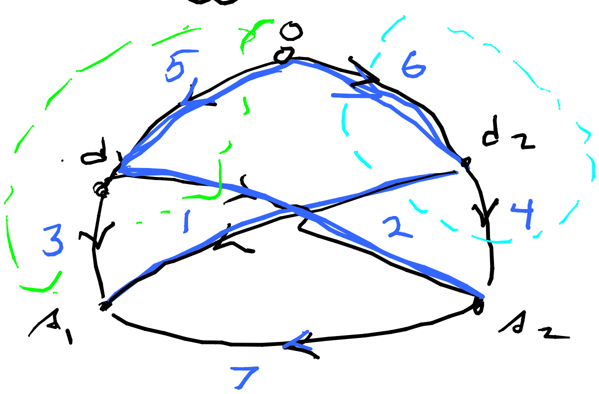
$$\beta = \frac{KP}{2}\frac{W}{L}, \quad V_{th} = VTO \qquad = \frac{2 I_O}{\sqrt{I_O/\beta}} = 2\sqrt{\beta I_O}$$



$$g_m v_{gs} = 2\sqrt{\beta I_0}$$



$\frac{v}{i} = z_{in}$ seen by the capacitor



$b = 7$

$n = 5$, $t = 4 = n - 1$

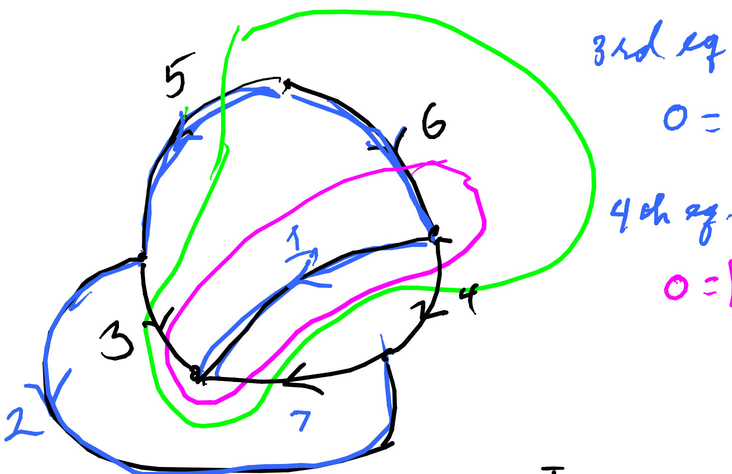
$\ell = 3$, $\ell = b - t$

Cut set eqs:
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix}_b \Rightarrow 0 = C\, i_b$$

Tie set eqs.
$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 1 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 & +1 & -1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix}_b \qquad 0 = T\, v_b$$

$$P = v_b^T i_b = v_t^T \mathcal{C}\,\mathcal{J}^T i_\ell = 0$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 1 \\ -1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$



3rd eq

$$0 = [0\ 0\ -1\ 1\ 1\ 0\ -1]\, i_b$$

4th eq.

$$0 = [0\ 0\ 1\ -1\ 0\ 1\ 1]\, i_b$$

$$v_b = \mathcal{C}^T v_t \quad , \quad i_b = \mathcal{J}^T i_\ell$$

Component equations: $\quad i_{1_b} = 0, \; i_{2_b} = 0$ as opens

$$\underline{i}_{(b)} = Y_{b\times b} \cdot \underline{v}_{(b)} \qquad i_{3_b} = g_m v_{1_b} \; , \; i_{4_b} = g_m v_{2_b}$$

$$G = 1/R \qquad i_{5_b} = G v_{5_b} \; ; \; i_{6_b} = G v_{6_b}$$

$$i_{b_7} = 0 + i$$

here

$$Y_{b\times b} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ g_m & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g_m & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} ; \quad j = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ i \end{bmatrix}$$

(7×7)

$v_b = v + e = 0$; $i_b = \underline{i} + j$; $0 = e\, i_b$

$= v$ $\qquad = Y_{b \times b} \cdot v + j$

$= e^T v_t$ $\qquad = \sigma^T i_\ell$

$$e\, i_b = 0 = \underbrace{e\, Y_{b \times b}\, e^T}_{t \times t} v_t + e j$$

t eqs in t unknowns, v_t ; 4 eqs in 4 unknowns

$$\underbrace{-e j}_{\text{equivalent sources in tree branches}} = \underbrace{(e\, Y_{b \times b}\, e^T)}_{\text{tree branch admittance}} \cdot v_t$$