Graph theory for eqs.

610_090804

KCL: $\underline{0} = C\, i_b$ $\quad t$ eqs.

(Σ of currents into a "sphere" [or out of] is 0, one eq. per tree branch)

KVL: $\underline{0} = \mho\, v_b$ $\quad \ell$ eqs.

(Σ of voltages around a loop in direction of cotree branch is 0)

t = # of tree branches

b = total # of branches

ℓ = # of link branches (C = cotree branches)

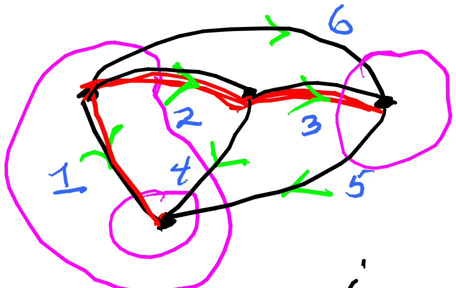
$b = t + \ell$

n = # of nodes in the graph

one separate part:

$t = n - 1 \;;\; \ell = b - t$
$\qquad\qquad\qquad\;\; = b - n + 1$

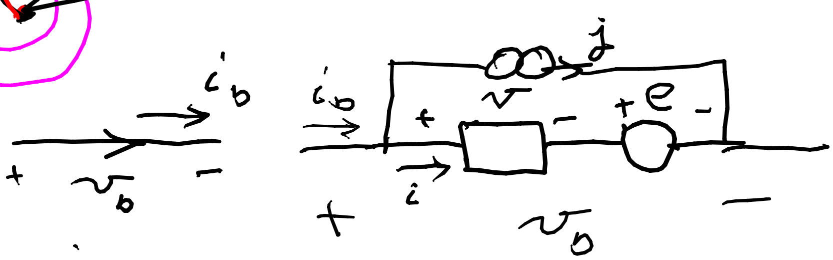
Ex:



$n = 4, \quad b = 6$

$t = 3, \quad \ell = 6 - 4 + 1 = 3$

choose



$i_b = i + j$

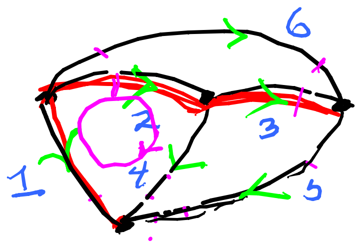
$v_b = v + e$

KCL: tree branch 1: $0 = i_{b_1} + i_{b_4} + i_{b_5}$

tree branch 2: $0 = i_{b_2} - i_{b_4} - i_{b_5} + i_{b_6}$

tree branch 3: $0 = i_{b_3} - i_{b_{\cancel{6}\,5}} + i_{b_6}$

$$\begin{bmatrix}0\\0\\0\end{bmatrix}=\left[\begin{array}{ccc|ccc}1&0&0&1&1&0\\0&1&0&-1&-1&1\\0&0&1&\underset{0}{\cancel{1}}&\underset{-1}{\cancel{1}}&1\end{array}\right]\left[\begin{array}{c}i_{b_1}\\i_{b_2}\\i_{b_3}\\\hline i_{b_4}\\i_{b_5}\\i_{b_6}\end{array}\right]=Q\,i_b=\left[\begin{array}{c|c}\underset{\underset{t}{\parallel}}{1_3}&K_i\end{array}\right]\begin{bmatrix}i_t\\i_\ell\end{bmatrix}$$



Tie set 1: $0 = v_{4_b} - v_{1_b} + v_{2_b}$
(branch 4)

(branch 5) 2 : $0 = v_{5_b} - v_{1_b} + v_{2_b} + v_{3_b}$

(branch 6) 3 : $0 = v_{6_b} - v_{2_b} - v_{3_b}$

$$\begin{bmatrix}0\\0\\0\end{bmatrix}=\left[\begin{array}{ccc|ccc}-1&1&0&1&0&0\\-1&1&1&0&1&0\\0&-1&-1&0&0&1\end{array}\right]\left[\begin{array}{c}v_{1b}\\v_{2b}\\v_{3b}\\\hline v_{4b}\\v_{5b}\\v_{6b}\end{array}\right]=T\,v_b=\left[\begin{array}{c|c}K_v&\underset{\underset{\ell}{\parallel}}{1_3}\end{array}\right]\begin{bmatrix}v_t\\v_\ell\end{bmatrix}$$

$$\Downarrow$$

$$1_3\cdot v_\ell = -K_v\, v_t$$

here $K_v = -K_i^T$ $\quad T$ = transpose

$$v_b=\begin{bmatrix}v_t\\v_\ell\end{bmatrix}=\begin{bmatrix}1_t\\-K_v\end{bmatrix}v_t=\begin{bmatrix}1_t\\K_i^T\end{bmatrix}v_t=Q^T v_t$$

also $i_b = T^T i_\ell$

as the graph is a closed system

$P_{in}(t)$ = total power into a sphere around it

$= 0$

$= v_{b_1} i_{b_1} + v_{b_2} i_{b_2} \cdots + v_{b_b} i_{b_b} = v_b^T i_b$

$$= v_t^T C \mathcal{J}^T i_\ell = 0 = \text{scalar zero}$$

holds for anything in the branches, so choose any voltage sources as v_b & any current sources as $i_\ell \Rightarrow$ can cancel v_t & i_ℓ from the above

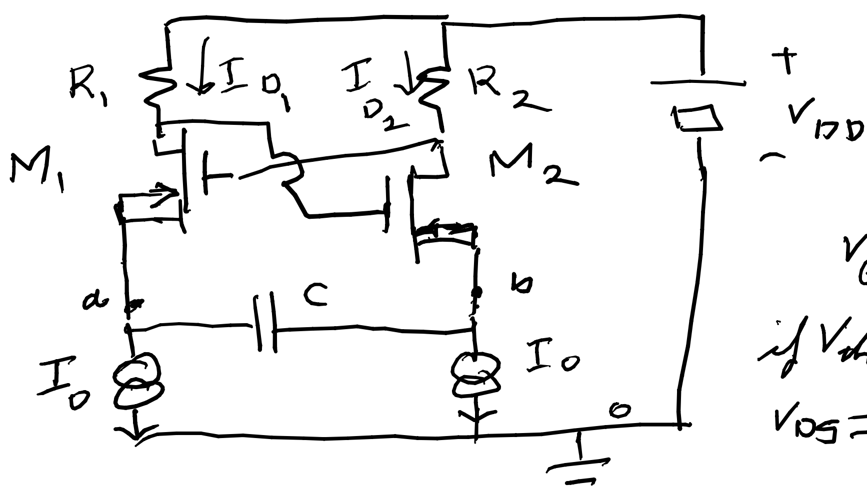
$$C \mathcal{J}^T = 0_{t \times \ell} = \left[1_t \mid K_i \right] \left[\begin{array}{c} K_v^T \\ \hline 1_\ell \end{array} \right]$$

by our nice numbering

$\Rightarrow K_v^T = -K_i$ will always be true

(KCL $\Leftrightarrow$ KVL)

another example: MOS multivibrator



at perfect bias $v_{a_0} = v_{b0}$

$\Rightarrow$

$V_{GS} = V_{DS}$ on M_1 & M_2

if $V_{th} > 0$ $VTO = V_{th}$

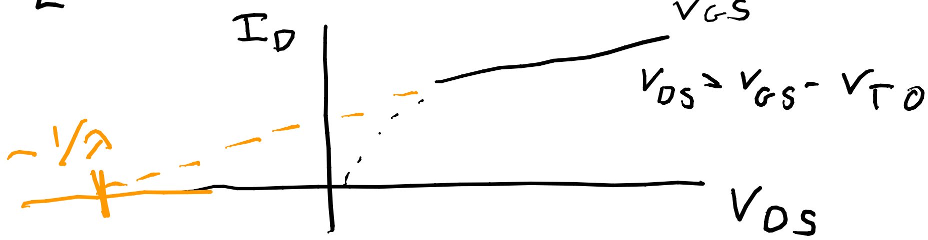
$V_{DS} = V_{GS} \geq V_{GS} - V_{th}$

law for transistors is

$$I_D = \frac{KP}{2} \cdot \frac{W}{L} (V_{GS} - VTO)^2 (1 + \lambda \cancel{V_{DS}})$$

V_{GS}

λ small



take $\lambda = 0$ for now; at bias

$$I_0 = I_D = \frac{KP}{2}\frac{W}{L}(V_{GS} - VTO)^2 \Rightarrow V_{GS} = VTO + \sqrt{\frac{I_0}{\frac{KP}{2}\frac{W}{L}}}$$

Now work with small change about the bias point

$$i_d = g_m v_{gs} \Rightarrow g_m = \left.\frac{dI_D}{dV_{GS}}\right|_{V_{GS}\ \text{bias}}$$

$$= \frac{KP}{2}\frac{W}{L} \cdot 2(V_{GS} - V_{TO}) = \left.\frac{2 I_0}{(V_{GS} - VTO)}\right|_{\text{bias}}$$

Ex: $4007 \Rightarrow KP = 20 \times 10^{-6}\ A/V^2$

$VTO = 1.3$