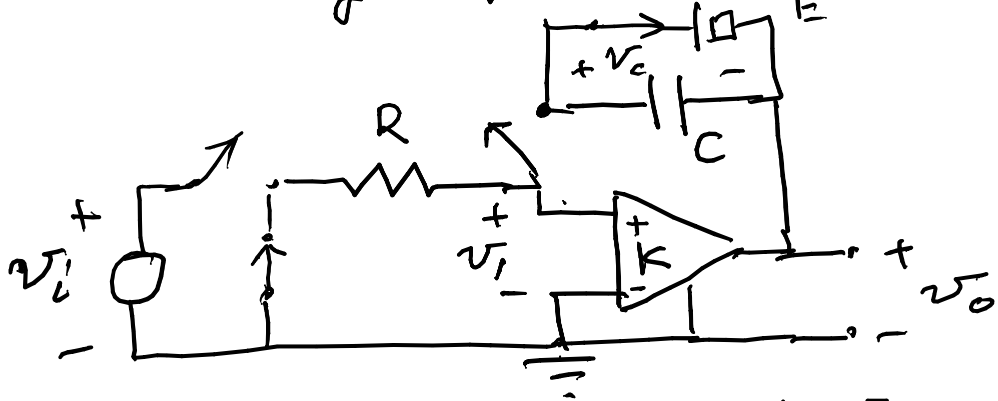
610_090104

Setting up initial conditions



Throw switches at $t = 0$

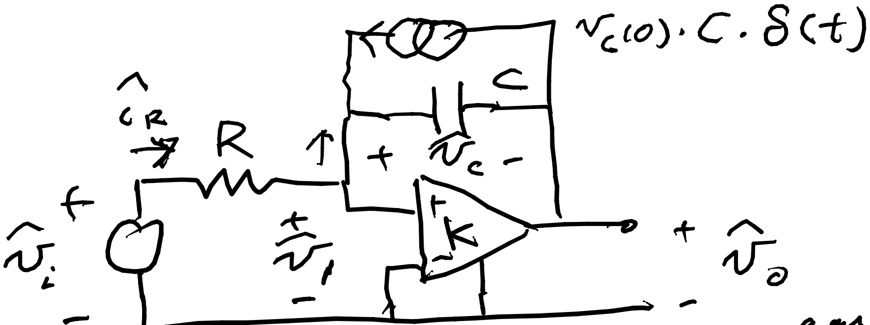
$v_c = E, \; t < 0$

$v_i = 0, \; v_o = 0$

$t < 0$

$$v_o = K v_1$$

this is a time-varying circuit



$1(t)$ = unit step function

use for $-\infty < t < \infty$
agrees with original for $t > 0$

let $\hat{v}_i = v_i 1(t)$, $\hat{v}_o(t) = v_o(t) 1(t)$, $\hat{v}_c(t) = v_c(t) 1(t)$

$$i_c(t) = C \frac{dv_c}{dt} \Rightarrow i_c \cdot 1 = C \frac{dv_c}{dt} 1 = \hat{i}_c = C\left\{\frac{d\hat{v}_c}{dt} - v_c(0)\delta(t)\right.$$

note $\dfrac{d(v_c 1)}{dt} = v_c(t) \dfrac{d1(t)}{dt} + \dfrac{dv_c}{dt} \cdot 1(t)$

$$\frac{dv_c}{dt} \cdot 1 = \frac{d\hat{v}_c}{dt} \cancel{+} - v_c(t)\delta(t) = \frac{d\hat{v}_c}{dt} \cancel{+} - v_c(0)\delta(t)$$

$$\hat{i}_R = G(\hat{v}_i - \hat{v}_1) = C\frac{d\hat{v}_c}{dt} - v_c(0)\,C\,\delta(t)$$

$$\hat{v}_c = \hat{v}_1 - \hat{v}_o, \quad \hat{v}_o = K\cdot\hat{v}_1$$

$$G(\hat{v}_i - G\cdot\tfrac{1}{K}\hat{v}_o) = C\frac{d}{dt}\left\{\tfrac{1}{K}\hat{v}_o - \hat{v}_o\right\} - v_c(0)\,C\,\delta(t)$$

$$\frac{d}{dt}\left(\frac{C}{K} - C + \frac{G}{K}\right)\cdot\hat{v}_o = -G\hat{v}_i - \frac{v_c(0)C}{G}\cdot G\,\delta(t)$$

$$\frac{(K-1)C+G}{K}\,\frac{d\hat{v}_o}{dt} = -G\left\{\hat{v}_i + \frac{v_c(0)C}{G}\,\delta(t)\right\}$$

for $-\infty < t < \infty$ only agrees with original circuit for $t > 0$.

Now assume $v_c(0) = 0$

$$\frac{(K-1)C+G}{K}\cdot\frac{d\hat{v}_o}{dt} = -G\hat{v}_i$$

if $K \to \infty$ have the op-amp

$$C\frac{d\hat{v}_o}{dt} = -G\hat{v}_i$$

Look at response to

$\hat{v}_i = V_i e^{st}$, $\quad -\infty < t < \infty$

assume $\hat{v}_o = V_o e^{st}$ $\quad$ " $\quad$ "

$$\frac{(K-1)C+G}{K}\,V_o\cdot s\cdot\cancel{e^{st}} = -G V_i \cancel{e^{st}}$$

$$\frac{V_o}{V_i} = \frac{-G}{\frac{(K-1)C+G}{K}\,s} = H(s) = \text{the transfer function}$$

here can interpret s:

a) derivative operator

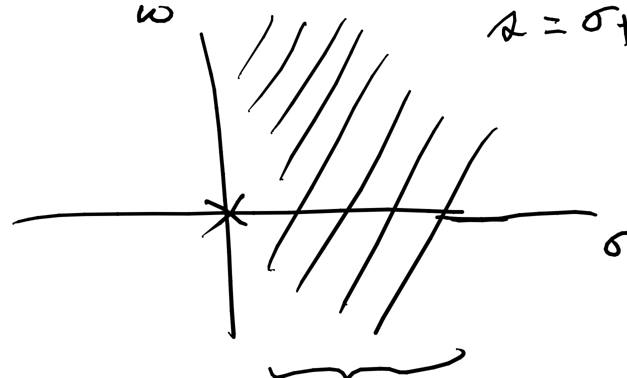
$v_o(t) = H(s)\cdot v_i(t)$

b) s in e^{st}

either a fixed complex number or a complex variable

c) s in Laplace transform (need the region of convergence)

Here $H(s) = \dfrac{-KG}{G+(K-1)C} \times \dfrac{1}{s}$



normal region of convergence for $\mathcal{L}(s) = \frac{\alpha}{s}$

use the bilateral Laplace transform

$$\mathcal{L}[f(t)] = \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt$$

Let $f(t) = 1(t)$

$$\mathcal{L}[1(t)] = \int_{-\infty}^{\infty} 1(t)\, e^{-st}\, dt = \int_{0}^{\infty} e^{-st}\, dt = \frac{1}{-s} e^{-st} \Big|_{t=0}^{t=\infty}$$

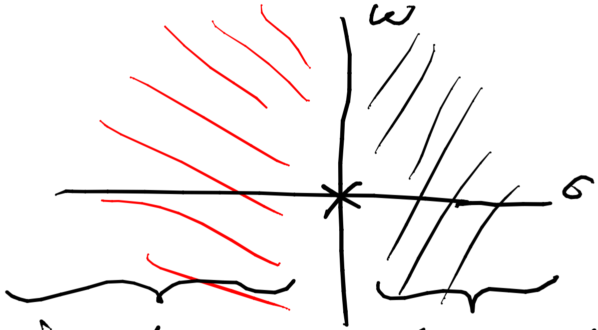
$$= -\frac{1}{s}\left[e^{-s(+\infty)} - e^{-s\times 0}\right]$$

need $\mathrm{Re}\, s > 0$ for $e^{-s\infty}$ to be finite & then it is zero so $\mathcal{L}[1(t)] = \frac{1}{s}$, $\sigma > 0$

$$\mathcal{L}[1(-t)] = \int_{-\infty}^{0} e^{-st}\, dt = -\frac{1}{s}\left[e^{-s\times 0} - e^{-s(-\infty)}\right]$$

needs $\mathrm{Re}\, s < 0$ then $\mathcal{L}[-1(-t)] = \frac{1}{s}$; in $\sigma < 0$

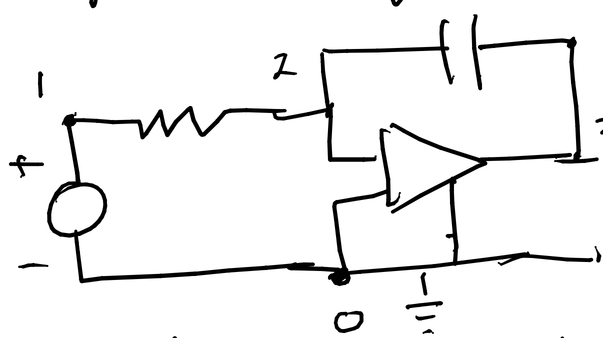
$\frac{1}{s}$



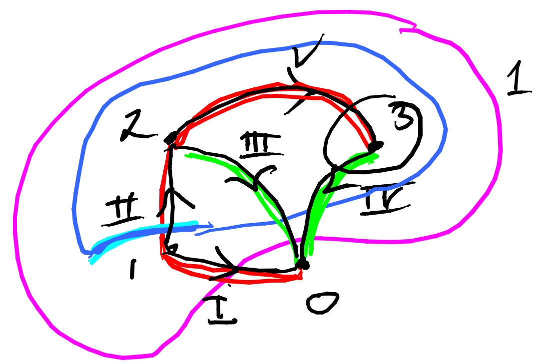
region of convergence of $\mathcal{L}[-1(-t)] = \frac{1}{s}$

Region of convergence of $\mathcal{L}[1(t)] = \frac{1}{s}$

Graph theory



$\Longrightarrow$



tree =

cotree =

KCL for cut 1: $i_{\mathrm{I}} + i_{\mathrm{III}} + i_{\mathrm{IV}} = 0$

" " 2: $i_{\mathrm{II}} - i_{\mathrm{III}} - i_{\mathrm{IV}} = 0$

" " 3: $i_{\mathrm{V}} - i_{\mathrm{IV}} = 0$