Problem 1

$$P(S) = S^6 + 3S^5 + 4S^4 + 6S^3 + 5S^2 + 3S + 2$$

5' a) $Z(S) = Ev\,P(S) / Od\,P(S) = \dfrac{S^6 + 4S^4 + 5S^2 + 2}{3S^5 + 6S^3 + 3S}$

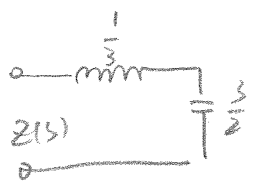
5' b)

$$
\begin{array}{r|l}
 & \tfrac{1}{3}S \\
3S^5+6S^3+3S & S^6+4S^4+5S^2+2 \\
 & \underline{S^6+2S^4+S^2} \\
 & 2S^4+4S^2+2
\end{array}
\qquad
\begin{array}{r|l}
 & \tfrac{3}{2}S \\
2S^4+4S^2+2 & 3S^5+6S^3+3S \\
 & \underline{3S^5+6S^3+3S} \\
 & 0
\end{array}
$$

$$Z(S) = \frac{1}{3}S + \frac{1}{\frac{3}{2}S} = \frac{S^2+2}{3S}$$

Positive Real and lossless

5' c)



15' d)

$$
\begin{array}{r|l}
 & S^4 \quad + 2S^2 \quad + 1 \\
S^2+3S+2 & S^6+3S^5+4S^4+6S^3+5S^2+3S+2 \\
 & \underline{S^6+3S^5+2S^4} \\
 & 2S^4+6S^3+5S^2+3S+2 \\
 & \underline{2S^4+6S^3+4S^2} \\
 & S^2+3S+2 \\
 & \underline{S^2+3S+2} \\
 & 0
\end{array}
$$

$P(S) = S^6 + 3S^5 + 4S^4 + 6S^3 + 5S^2 + 3S + 2$

$P_1(S) = S^2 + 3S + 2$

5' if conclude that $P(S)$ is HP.

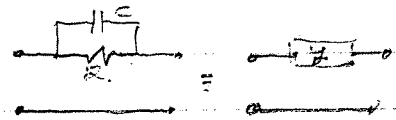
$\therefore\ P(S) = P_1(S) \cdot P_2(S)$

$Z(S) = \dfrac{S^2+2}{3S}$ is a reactance function $\Rightarrow$ $P_1(S)$ is Hurwitz polynomial

$P_2(S) = S^4 + 2S^2 + 1$ clearly not a Hurwitz 1st and 3rd order cofactor $= 0$

$\therefore\ P(S) = P_1(S) \cdot P_2(S)$ not hurwitz.

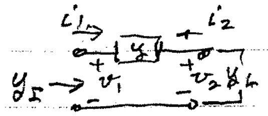
#2



$G = 1/R$

$y(s) = Cs + G$, $\quad Y(s) = y(s)\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$

$\Delta Y \equiv 0$



$$y_i = \frac{1}{z + z_L} = \frac{1}{\frac{1}{y} + \frac{1}{y_L}} = \frac{y\,y_L}{y + y_L}$$

also found by $i_2 = -y_L v_2 = y_{21} v_1 + y_{22} v_2 \Rightarrow (y_L + y_{22}) v_2 = -y_{21} v_1$

$$i_1 = y_{11} v_1 + y_{12} v_2 = \left(y_{11} - \frac{y_{12} y_{21}}{y_L + y_{22}}\right) v_1 = \frac{\Delta Y + y_{11} y_L}{y_{22} + y_L} v_1$$

$\therefore\ y_i = \dfrac{\Delta Y + y_{11} y_L}{y_{22} + y_L}$ where $\Delta Y = 0$ here

$\Rightarrow y_i y_{22} + y_i y_L = \Delta Y + y_{11} y_L \Rightarrow (y_i - y_{11}) y_L = \Delta Y - y_{22} y_i$

$\Rightarrow y_L = \dfrac{\Delta Y - y_{22} y_i}{y_i - y_{11}}$ $\quad\therefore\ y_i = \dfrac{y\,y_L}{y + y_L}$ & $y_L = \dfrac{y\,y_i}{y - y_i}$

$$y_i = \frac{(sC+G)\,y_L}{(sC+G) + y_L}\ ; \quad y_L = \frac{(sC+G)\,y_i}{(sC+G) - y_i} = \frac{1}{z_i - \frac{1}{sC+G}}$$

$$\Rightarrow z_L = z_i - \frac{1}{sC+G}$$

for $y_L = PR \Rightarrow z_L = PR \Rightarrow \exists$ pole in z_i @ $s = -C/G$

$\therefore$ for degree reduction $\frac{1}{y_i} = z_i$ has a simple pole at $s_0 = \sigma_0 \le 0$ necessarily must choose $C/G = \sigma_0$ for degree reduction (if at $\sigma_0 = 0$, $G = 0$)

If $z_i = \dfrac{1}{sC+G} + z_2$ then, as degree reduction was forced, z_2 must be PR

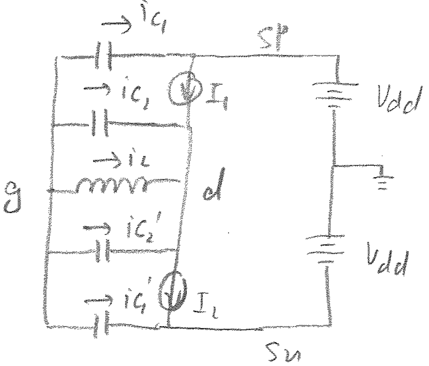
$\therefore$ necessary & sufficient

$$\frac{1}{y_i} = \frac{k_0}{s - s_0} + z_2, \quad s_0 \le 0,\ k_0 > 0,\ z_2 = PR$$

& no pole @ s_0

problem 3

$x = [i_L, V_g, V_L]^T$



20' a) state variable equation

$$v_L = L\frac{di_L}{dt} \Rightarrow \frac{di_L}{dt} = \frac{1}{L}V_L \quad \cdots\cdots ①$$

$$I_1 + i_{C_2} + i_L + i_{C_2}' - I_2 = 0$$

$$i_{C_2} = i_{C_2}' = C_2\frac{dv_L}{dt}$$

$$\therefore\ 2C_2\frac{dv_L}{dt} = I_2 - I_1 - i_L$$

$$I_2 - I_1 = \beta(V_g + V_{dd} - V_{T0})^2 - \beta(V_g - V_{dd} - V_{T0})$$

$$= \beta(2V_g - 2V_{T0})(2V_{dd}) = 4\beta V_{dd}(V_g - V_{T0})$$

$$\therefore\ 2C_2\frac{dv_L}{dt} = 4\beta V_{dd}(V_g - V_{T0}) - i_L$$

$$\therefore\ \frac{dV_L}{dt} = -\frac{1}{2C_2}i_L + \frac{2\beta V_{dd}}{C_2}V_g - \frac{2\beta V_{dd}V_{T0}}{C_2} \quad \cdots\ ②$$

$$\left.\begin{aligned} i_{C_1} + i_{C_2} + i_L + i_{C_2}' + i_{C_1}' &= 0 \\ I_1 + i_{C_2} + i_L + i_{C_2}' - I_2 &= 0 \end{aligned}\right\} \Rightarrow i_{C_1} + i_{C_1}' = I_1 - I_2$$

$$i_{C_1} = i_{C_1}' = C_1\frac{dV_g}{dt}$$

$$\therefore\ \frac{dV_g}{dt} = -\frac{2\beta V_{dd}}{C_1}V_g + \frac{2\beta V_{dd}V_{T0}}{C_1} \quad \cdots\cdots ③$$

$$\therefore\ \boxed{\begin{aligned} \frac{di_L}{dt} &= \frac{1}{L}V_L \\ \frac{dV_g}{dt} &= -\frac{2\beta V_{dd}}{C_1}V_g + \frac{2\beta V_{dd}V_{T0}}{C_1} \\ \frac{dV_L}{dt} &= -\frac{1}{2C_2}i_L + \frac{2\beta V_{dd}}{C_2}V_g - \frac{2\beta V_{dd}V_{T0}}{C_2} \end{aligned}}$$

5' for each equation

3 for 20'

$$\frac{d}{dt}\begin{bmatrix} i_L \\ V_g \\ V_L \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{1}{L} \\ 0 & -\frac{2\beta V_{dd}}{C_1} & 0 \\ -\frac{1}{2C_2} & \frac{2\beta V_{dd}}{C_2} & 0 \end{bmatrix}\begin{bmatrix} i_L \\ V_g \\ V_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2\beta V_{dd}V_{T0}}{C_1} \\ -\frac{2\beta V_{dd}V_{T0}}{C_1} \end{bmatrix}$$

②

Problem 3

10' b) linearize the state variable equation

$$\frac{d}{dt}\begin{bmatrix} i_L \\ v_g \\ v_c \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{1}{L} \\ 0 & -\frac{2\beta V_{dd}}{C_1} & 0 \\ -\frac{1}{2C_1} & \frac{2\beta V_{dd}}{C_2} & 0 \end{bmatrix}\begin{bmatrix} i_L \\ v_g \\ v_c \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & \frac{1}{L} \\ 0 & -\frac{2\beta V_{dd}}{C_1} & 0 \\ -\frac{1}{2C_2} & \frac{2\beta V_{dd}}{C_2} & 0 \end{bmatrix}$$

$$\det(s\mathbb{1}_3 - A) = \left|\begin{bmatrix} s & 0 & -\frac{1}{L} \\ 0 & s+\frac{2\beta V_{dd}}{C_1} & 0 \\ \frac{1}{2C_2} & -\frac{2\beta V_{dd}}{C_2} & s \end{bmatrix}\right| = s^2\left(s+\frac{2\beta V_{dd}}{C_1}\right) + \frac{1}{2LC_2}\left(s+\frac{2\beta V_{dd}}{C_1}\right)$$

+5' for only this without solving $s_{1,2,3}$

$$\therefore \det(s\mathbb{1}_3 - A) = \left(s^2 + \frac{1}{2LC_2}\right)\left(s+\frac{2\beta V_{dd}}{C_1}\right)$$

Natural frequency: $s_{12} = \pm j/\sqrt{2LC_2}$ $\quad s_3 = -\frac{2\beta V_{dd}}{C_1}$

5' c) Oscillation frequency:

$$\omega = \frac{1}{\sqrt{2LC_2}}$$

$$\text{or } f = \frac{1}{2\pi\sqrt{2LC_2}}$$