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ENEE 610

Problems to consider, Set 5

The van der Pol Oscillator and Some Scattering Matrices

1. For the van der Pol oscillator of class, written up in the file vanderPol1.pdf, set up and run the corresponding Spice file.
2. Redo 1. for the revised van der Pol oscillator

$$\frac{d^2x}{dt^2} + \varepsilon(3x^2 - 1)\frac{dx}{dt} + \omega_o^2 x = 0; \qquad x(0) \text{ and } \frac{dx}{dt}(0) \text{ given}$$

3. a) Set up a van der Pol oscillator

$$\frac{dx}{dt} = y + \varepsilon f(x) \qquad \text{given } x(0)$$

$$\frac{dy}{dt} = -\omega_o^2 x \qquad \text{given } y(0)$$

using the piecewise approximation to the cubic nonlinearity $x - x^3/3$:

$$f(x) = \begin{cases} -2x - 3 & x < -1 \\ x & -1 \le x \le 1 \\ -2x + 3 & 1 < x \end{cases}$$

b) Plot f(x) and compare with $x - x^3/3$.

c) Write f(x) in the form

$$f(x) = a_o + a_\infty x + \sum_i a_i \,|x - b_i|$$

4. a) Check that the impedance matrix for a lossless transmission line of length l and inductance L and capacitance C per unit length, considered as a 2-port, is given by

$$Z(s) = \sqrt{\frac{L}{C}} \begin{bmatrix} \operatorname{ctnh}(\sqrt{LC}ls) & \operatorname{csch}(\sqrt{LC}ls) \\ \operatorname{csch}(\sqrt{LC}ls) & \operatorname{ctnh}(\sqrt{LC}ls) \end{bmatrix}$$

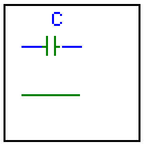
b) Find the admittance matrix.

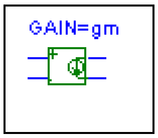
c) Find the scattering matrix.

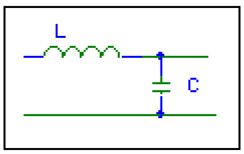
d) Give the partial fraction expansion for the admittance matrix and from this give the location of the poles.

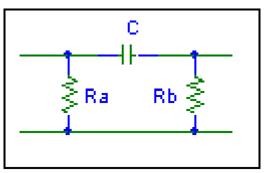
e) (challenging) Consider putting an infinite number of these in parallel where the lengths cover all rational numbers between zero and one (keep L and C the same. Show that this puts a singularity in the resulting Y(s) at every imaginary number since the irrationals are limits of rationals. What does this mean in terms of the positive realness? What happens to bounded realness (especially S(s) should have no imaginary poles!)?

5. Find the scattering matrix for the following 2-ports (port 1 on the left):

a) 

b) 

c) 

d) 

e) The parallel combination of b) and d) which is essentially the pi model of a transistor.

5. Find the scattering matrix for the following 2-port (port 1 on the left)