file: c:\math\mcad80\rwn_mcd\richards1.mcd RWN 09/10/03 corrected 09/16-17/03

Example for synthesis using Richards'' function

$k := 2$ $a := \frac{1}{3}$ $b := 1$ $c := 2$ $d := 3$

$z(s) := \frac{((s+a)\cdot(s+c))}{((s+b)\cdot(s+d))}$ $Nz(s) := s^2 + (a+c)\cdot s + a\cdot c$ $Dz(s) := s^2 + (b+d)\cdot s + b\cdot d$

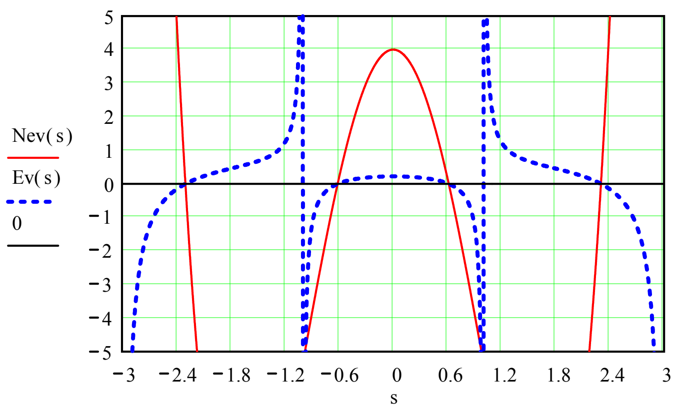
$Ev(s) := \frac{(z(s)+z(-s))}{2}$ $Nev(s) := Nz(s)\cdot Dz(-s) + Nz(-s)\cdot Dz(s)$

To plot to see where zeros of even part may be

$smax := 3$ $sinc := 0.001$ $smin := -smax$

$Emax := 5$ $Emin := -Emax$

$s := smin, smin + sinc .. smax$



To solve more precisely for zeros of the even part use trials at s=0.6 and s=2.0

$s := 0.6$

$r1 := root(Nev(s), s)$ $r1 = 0.61496343$ $r1^2 = 0.37818002$

$s := 23$

$r2 := root(Nev(s), s)$

$r2 = 2.29967099$

$k := r1$ $z(k) = 0.42476$ $Nev(r1) = 9.97763205\cdot10^{-10}$ $Ev(r1) = 9.30537314\cdot10^{-11}$

$Nev(-r1) = 9.97763205\cdot10^{-10}$ $Nev(r2) = 1.25598234\cdot10^{-8}$

More precise evaluation in this case:

$B := b\cdot d + a\cdot c - (a+c)\cdot(b+d)$ $B = -5.66666667$

$C := a\cdot b\cdot c\cdot d$ $C = 2$

$$r1 := \sqrt{\frac{-B}{2} - \left(\frac{1}{2}\right)\cdot\sqrt{\left(B^2 - 4\cdot C\right.}}$$ $r1 = 0.61496343$

$$r2 := \sqrt{\left(\frac{-B}{2}\right) + \left(\frac{1}{2}\right)\cdot\sqrt{\left(B^2 - 4\cdot C\right.}}$$ $r2 = 2.29967099$

$k := r1$

For the richards function $R(s,k) := \frac{(k \cdot z(s) - s \cdot z(k))}{(k \cdot z(k) - s \cdot z(s))}$ we are interested in

$Nr(s) := (k \cdot (s + a) \cdot (s + c) - s \cdot z(k) \cdot (s + b) \cdot (s + d))$

$Nr(s) := -z(k) \cdot s^3 + (k - (b + d) \cdot z(k)) \cdot s^2 + (k \cdot (a + c) - z(k) \cdot b \cdot d) \cdot s + k \cdot a \cdot c$

$n3 := -z(k)$ $\quad n3 = -0.42475968$ $\quad n0 := k \cdot a \cdot c$

$n2 := k - (b + d) \cdot z(k)$ $\quad n2 = -1.0840753$ $\quad n0 = 0.40997562$

$n1 := k \cdot (a + c) - z(k) \cdot b \cdot d$ $\quad n1 = 0.16063563$

$Nr(s) := -0.42475968 \cdot s^3 + -1.0840753 \cdot s^2 + 0.16063563 \cdot s + 0.40997562$

$\frac{n2}{n3} = 2.55220856$ $\quad \frac{n1}{n3} = -0.37818002$ $\quad \frac{n0}{n3} = -0.96519429$

$Nr(s) := -0.42475968 \cdot \left(s^3 + 2.55220856 \cdot s^2 + -0.37818002 \cdot s + -0.96519429\right)$

$NRi(s) := \frac{Nr(s)}{s^2 - 0.37818002}$ $\quad zeronum := -2.55220856$

$zeronum = -2.55220856$

$Dr(s) := k \cdot z(k) \cdot (s + b) \cdot (s + d) - s \cdot (s + a) \cdot (s + c)$

$Dr(s) := -s^3 + (k \cdot z(k) - (a + c)) \cdot s^2 + (k \cdot z(k) \cdot (b + d) - a \cdot c) \cdot s + k \cdot z(k) \cdot b \cdot d$

$d3 := -1$ $\quad d0 := k \cdot z(k) \cdot b \cdot d$

$d2 := k \cdot z(k) - (a + c)$ $\quad d2 = -2.07212166$ $\quad d0 = 0.78363502$

$d1 := k \cdot z(k) \cdot (b + d) - a \cdot c$ $\quad d1 = 0.37818002$

$Dr(s) := -1 \cdot s^3 + -2.07212166 \cdot s^2 + 0.37818002 \cdot s + 0.78363502$

$Dr(s) := -1 \cdot \left(s^3 + 2.07212166 \cdot s^2 - 0.37818002 \cdot s - 0.78363502\right)$

$zerodem := -2.07212166$

$zerodem = -2.07212166$

$Nr(r1) = -1.45271449 \cdot 10^{-9}$ $\quad Nr(-r1) = 5.741674 \cdot 10^{-11}$ $\quad Nr(r2) = -10.11956648$

$Dr(r1) = 1.41140455 \cdot 10^{-9}$ $\quad Dr(-r1) = 4.96666508 \cdot 10^{-9}$ $\quad Dr(r2) = -21.46684238$

Therefore $R(s,k) := \frac{(k \cdot z(s) - s \cdot z(k))}{(k \cdot z(k) - s \cdot z(s))} = \left(\frac{n3}{d3}\right)$ (s-zerinum)/(s-zerodem))

$R(s,k) := 0.42475968 \cdot \frac{\left[\left(s^2 - 0.37818002\right) \cdot (s + 2.55220856)\right]}{\left[\left(s^2 - 0.37818002\right) \cdot (s + 2.07212166)\right]}$ corrected zeronum

$rgyr1 := z(k)$ $\quad rgyr1 = 0.42475968$ $\quad C1 := \frac{1}{k \cdot z(k)}$ $\quad C1 = 3.82831284$

$$KL := \left(\frac{n3}{d3}\right) \cdot z(k) \qquad KL = 0.18042079$$

$$e := -\,zeronum \qquad f := -\,zerodem$$

$$zL(s) := KL \cdot \frac{(s+e)}{(s+f)}$$

zL(s) is the load on the first lossless coupling section. To repeat on it.

$$EvzL(s) := \frac{(zL(s) + zL(-s))}{2} \qquad EvzL(r2) = 1.0451348 \cdot 10^{-9}$$

$$NzLEv(s) := KL \cdot \frac{((s+e)\,(-s+f) + (-s+e)\,(s+f))}{2}$$

$$NzLEv(s) := KL \cdot \left(-s^2 + e \cdot f\right) \qquad e \cdot f = 5.28848664$$

$$r3 := \sqrt{e \cdot f} \qquad r3 = 2.29967098$$

$$zL(r3) = 0.20023363 \qquad zL(-r3) = -0.20023363$$

$$EvzL(r3) = 0$$

Forming Richards' function of zL(s): $RL(s,k) := \frac{(k \cdot zL(s) - s \cdot zL(k))}{(k \cdot zL(k) - s \cdot zL(s))}$

In the following KL cancels numerator and denominator so is dropped

$$k2 := r3$$

$$NRL(s) := k2 \cdot (s+e) \cdot (k2+f) - s \cdot (k2+e) \cdot (s+f)$$

$$NRL(s) := -(k2+e) \cdot s^2 + -((k2 \cdot (k2+f) - (k2+e) \cdot f) \cdot s + k2 \cdot e \cdot (k2+f)$$

$$nR2 := -(k2+e) \qquad nR2 = -4.85187954$$

$$nR1 := (k2 \cdot (k2+f) - (k2+e) \cdot f) \qquad nR1 = 0$$

$$nR0 := k2 \cdot e \cdot (k2+f) \qquad nR0 = 25.65910014$$

$$\frac{nR0}{nR2} = -5.28848664$$

$$DRL(s) := k2 \cdot (k2+e) \cdot (s+f) - s \cdot (k2+f) \cdot (s+e)$$

$$DRL(s) := -(k2+f) \cdot s^2 + (k2 \cdot (k2+e) - e \cdot (k2+f)) \cdot s + (k2 \cdot (k2+e) \cdot f$$

$$dR2 := -(k2+f) \qquad dR2 = -4.37179264$$

$$dR1 := k2 \cdot (k2+e) - e \cdot (k2+f) \qquad dR1 = 0$$

$$dR0 := (k2 \cdot (k2+e) \cdot f \qquad dR0 = 23.12016699$$

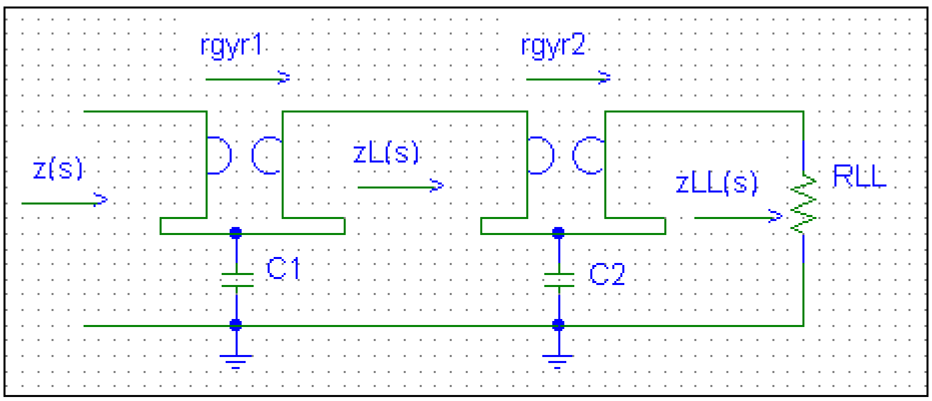
$$\frac{dR0}{dR2} = -5.28848664$$

$$zLL(s) := zL(k2)\cdot\frac{\left[KL\cdot nR2\cdot\left(s^2 - \frac{nR0}{nR2}\right)\right]}{\left[KL\cdot dR2\cdot\left(s^2 - \frac{dR0}{dR2}\right)\right]}$$

$$zLL(s) := zL(k2)\cdot\frac{nR2}{dR2} \qquad zLL(s) = 0.22222222$$

$rgyr2 := zL(k2)$ $\qquad rgyr2 = 0.20023363$

$C2 := \frac{1}{k2\cdot zL(k2)}$ $\qquad C2 = 2.17168716$ $\qquad$ C2 corrected with zL 09/17/03



$k1 := 0.61496343$ $\qquad k2 = 2.29967098$

$$z(s) := \frac{\left[\left(s + \frac{1}{3}\right)(s+2)\right]}{(s+1)\cdot(s+3)} \qquad zL(s) := 0.18042079\cdot\frac{(s + 2.55220856)}{(s + 2.07212166)} \qquad RLL := 0.22222222$$

$rgyr1 = 0.42475968$ $\qquad rgyr2 = 0.20023363$

$C1 = 3.82831284$ $\qquad C2 = 2.17168716$

Note that an alternate sets of element values can be obtained by extracting