

Positive Real Functions and Circuit Graphs

1. (25 points)

We know that a lossless passive finite circuit input admittance $Y(s)$ is a rational positive real function which is odd in s , $Y(s) = -Y(-s)$.

- Show that $sY(s)$ is a function $y(\cdot)$ of s^2 , $y(s^2) = sY(s)$.
- Show that $y(s)$ as an admittance is necessarily positive real. Is it realizable by a two element kind circuit (that is, is it realizable as one of LC, RC, LR)?

2. (10 points)

We also know that an LC realization of the $Y(s)$ of 1. can be turned into an RC circuit by replacing each L by an R. Explain why we can not do the same when we synthesize $Y(s)$ using a cascade of Richards' sections. Explain also why an arbitrary rational positive real function can not be synthesized by a cascade of degree one real Richards' sections (hint type of note: we can use degree two real sections whenever degree one sections will not work).

3. (35 points)

For the following circuit

- Draw a graph for the circuit numbering nodes as given and branches by the numbers of the elements while orienting all branches down.
- Give the cutset, tieset and branch by branch admittance matrices.
- From the resulting equations calculate the input impedance seen by the current source.

