Solution EE610 final 12/18/03

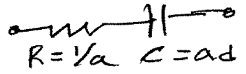
#1. as the other coefficients are >0, n and d must be ≥ 0 to guarantee no poles or zeros in $\operatorname{Re} s > 0$. If $d=0$ then $n=0$ as poles at ∞ must be simple. If $d>0$ any pole at ∞, of residue n/d, can be removed and the remainder will be PR. By long division

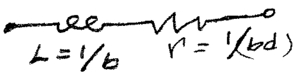
$$s + 1/d \,\overline{\big)\, \frac{n}{d}s^2 + \frac{3}{d}s + \frac{1}{d}}\quad \overset{\frac{n}{d}s}{} \qquad \frac{n}{d}s^2 + \frac{n}{d^2}s \quad \Rightarrow y(s) = \frac{n}{d}s + \frac{\frac{3d-n}{d^2}s + 1/d}{s + 1/d}$$

To avoid a zero in $\operatorname{Re} s > 0$ we need $3d - n \geq 0 \Rightarrow 3d \geq n \geq 0$

Note that $\frac{as+b}{s+1/d} = \frac{as}{s+1/d} + \frac{b}{s+1/d}$ is PR if $a \geq 0$ & $b \geq 0$ when $d>0$

because $\frac{as}{s+1/d} = \begin{cases} 0 \text{ if } a=0 \\ \frac{1}{\frac{1}{a}+\frac{1}{ads}} \text{ if } a \neq 0 \end{cases}$ $\Rightarrow$ open

$\Rightarrow$ 

$\frac{b}{s+1/d} = \begin{cases} 0 \text{ if } b=0 \Rightarrow \text{open (as } b = 1/d \text{ not possible here)} \\ \frac{1}{\frac{s}{b}+\frac{1}{bd}} \text{ if } b \neq 0 \Rightarrow \end{cases}$ 

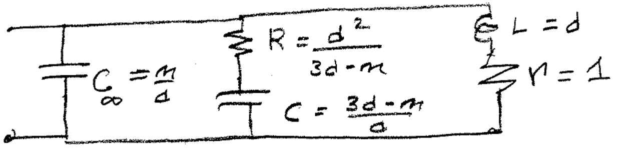
$\therefore\; y(s) = \frac{ns^2+3s+1}{ds+1}$ is PR if $\begin{cases} d=0 \text{ then } n=0 \\ d \neq 0 \text{ then } 3d \geq n \geq 0 \end{cases}$

b) $y(s)$ is lossless if $y(s) = -y(-s) \Rightarrow \frac{ns^2+3s+1}{ds+1} = -\left(\frac{ns^2-3s+1}{-ds+1}\right)$

$\Rightarrow (ns^2+3s+1)(-ds+1) = (ns^2-3s+1)(-ds-1)$

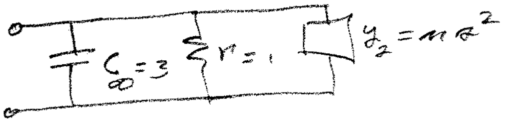
at $s=0 \Rightarrow 1 = -1$ or <u>can never be lossless</u>

c) Synthesis; PR case

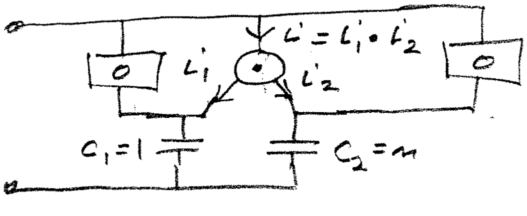


with opens, C_∞ if $n=0$, R if $3d=n$

d) If $d \neq 0$ the circuit of c) holds with possibly negative elements.

If $d=0$, $y(s) = ns^2+3s+1 \Rightarrow$ 

For $y_2 = ns^2$, $n \neq 0$, we can use a current multiplier and a two nullators (to apply v onto the capacitor)



Note that a current multiplier is active as needed for a non-PR function

#1. c) alternate $y(s) - \frac{n}{d}s = \frac{\frac{3d-n}{d^2}s + 1/d}{s + 1/d} = y_1(s)$

$$y_1(j\omega) = \frac{(1/d + j\frac{3d-n}{d^2}\omega)(1/d - j\omega)}{((\frac{1}{d})^2 + \omega^2)} \Rightarrow \text{Re}\, y_1(j\omega) = \frac{(1/d)^2 + \frac{3d-n}{d^2}\omega^2}{(1/d)^2 + \omega^2} = R(\omega^2)$$

Find $\min_{\omega} \text{Re}\, y_1(j\omega)$ as can subtract this out as a resistor

$$\frac{d}{d\omega} R(\omega^2) = \frac{dR(\omega^2)}{d\omega^2} \cdot \frac{d\omega^2}{d\omega} = 2\omega \frac{dR(\omega^2)}{d\omega^2};$$

$$\frac{dR(\omega^2)}{d\omega^2} = \frac{\frac{3d-n}{d^2}}{(1/d)^2 + \omega^2} - \frac{[(1/d)^2 + \frac{3d-n}{d^2}\omega^2]}{((1/d)^2 + \omega^2)^2} = \frac{\frac{3d-n}{d^2}(1/d^2) - (1/d)^2}{((1/d^2) + \omega^2)^2}$$

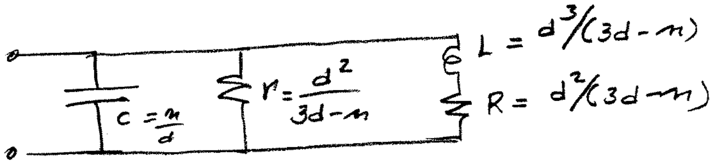
$$= \frac{(1/d)^2\left[\frac{3d-n}{d^2} - 1\right]}{((1/d)^2 + \omega^2)^2} = 0 \text{ only @ } \omega = \infty \text{ if } 3d - n - d^2 \neq 0$$

$$\frac{dR(\omega^2)}{d\omega} = \frac{2\omega\left[\frac{3d-n-d^2}{d^4}\right]}{((1/d)^2 + \omega^2)^2} = 0 \Rightarrow \omega = \infty$$

$$\therefore\ y_2 = y_1 - \frac{3d-n}{d^2} = \frac{\frac{3d-n}{d^2}s + 1/d}{s + 1/d} - \frac{3d-n}{d^2} = \frac{(1/d)\left[\frac{3d-n}{d^2}\right]}{s + 1/d}$$

$$\therefore\ y = \frac{n}{d}s + \frac{3d-n}{d^2} + \frac{\frac{3d-n}{d^3}}{s + 1/d} = \frac{n}{d}s + \frac{3d-n}{d^2} + \frac{1}{\frac{d^3}{3d-n}s + \frac{d^2}{3d-n}}$$

$\Rightarrow y \rightarrow$



Note that this uses the minimum number of reactive elements

#2. a) tree branch 1: $0 = i_1 + i_4 + i_6$

branch 2: $0 = i_2 - i_4 + i_5 - i_6$

branch 3: $0 = i_3 + i_4 - i_5$

$$\left.\begin{array}{l} \\ \\ \end{array}\right\} 0 = \mathcal{C} i_b \Rightarrow \mathcal{C} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{array}\right]$$

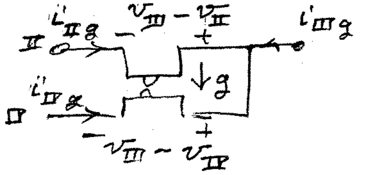
b) link branch 4: $0 = v_4 - v_1 + v_2 - v_3$

branch 5: $0 = v_5 - v_2 + v_3$

branch 6: $0 = v_6 - v_1 + v_2$

$$\left.\begin{array}{l} \\ \\ \end{array}\right\} 0 = \mathcal{T} v_b \Rightarrow \mathcal{T} = \left[\begin{array}{ccc|ccc} -1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

c) Sum currents out of nodes I, II, III, IV. Only the gyrator causes concern



$$-i_{IIg} = -g(v_{III} - v_{IV})$$

$$-i_{IVg} = g(v_{III} - v_{II})$$

$$i_{IIIg} = -(i_{IIg} + i_{IVg}) =$$

$$= g(v_{IV} - v_{II})$$

Use $R = 1/G$

Then
$$Y_{ind} = \begin{array}{c} \mathrm{I} \\ \mathrm{II} \\ \mathrm{III} \\ \mathrm{IV} \\ \\ \end{array} \begin{array}{c} \left[\begin{array}{cccc} s(C_2+C_5) & -sC_2 & -sC_5 & 0 \\ -sC_2 & sC_2+G & g & -G-g \\ -sC_5 & -g & sC_5 & g \\ 0 & -G+g & -g & G \end{array}\right] \\ \begin{array}{cccc} \mathrm{I} & \mathrm{II} & \mathrm{III} & \mathrm{IV} \end{array} \end{array}$$

note rows & columns all add to 0

d) scratch out row & column IV

$$Y = \begin{bmatrix} s(C_2+C_5) & -sC_2 & -sC_5 \\ -sC_2 & sC_2+G & g \\ -sC_5 & -g & sC_5 \end{bmatrix} \Rightarrow \begin{bmatrix} I_I \\ I_{II} \\ I_{III} \end{bmatrix} = Y \begin{bmatrix} V_I \\ V_{II} \\ V_{III} \end{bmatrix} \Rightarrow \begin{bmatrix} I_I \\ 0 \\ 0 \end{bmatrix} = Y \begin{bmatrix} V_{in} \\ v_{II} \\ v_{III} \end{bmatrix}$$

e)
$$\Rightarrow \begin{bmatrix} sC_2 \\ sC_5 \end{bmatrix} V_{in} = \begin{bmatrix} sC_2+G & g \\ -g & sC_5 \end{bmatrix} \begin{bmatrix} v_{II} \\ v_{III} \end{bmatrix} \Rightarrow \begin{bmatrix} v_{II} \\ v_{III} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} sC_5 & -g \\ g & sC_2+G \end{bmatrix} \begin{bmatrix} C_2 \\ C_5 \end{bmatrix}$$

where $\Delta = (sC_2+G)sC_5 + g^2 =$ determinant $\neq 0$ as $C > 0$

$$\therefore\ V_o = v_{II} - v_{III} = \left[\left(\cancel{sC_5 \cdot sC_2} - g\,sC_5\right) - \left(g\,sC_2 + \cancel{sC_2 \cdot sC_5} + sC_5 G\right)\right] \frac{V_{in}}{\Delta}$$

$$\therefore\ \frac{V_o}{V_{in}} = \frac{-[g(C_2+C_5) + GC_5]s}{C_2C_5 s^2 + GC_5 s + g^2} = \frac{-[2g+G]Cs}{C^2s^2 + GCs + g^2}$$

if $C_2 = C_5 =$

$$i_3 = -g\,v_{III} = -g[gC_2 s + C_2C_5 s^2 + GC_5 s]\,V_{in}/\Delta$$

if $C_2 = C_5 = C$

$$= -g[C^2s^2 + (g+G)Cs]\,V_{in}/\Delta$$

$$= -gCs[Cs + (g+G)]\,V_{in}/\Delta$$

f) for balance

both V_o & $i_3 = 0$ @ $s = 0$ or $g = G = 0$ or $g = -\frac{G}{2}$ @ $s = -\frac{g+G}{C} = \frac{-G}{2C}$

Can use to detect g (or G) @ $s = -\frac{G}{2C}$ for $g = -\frac{G}{2}$

#3. II.

$$i_D = f(v_{in} - v_2, v_1) = f(u - x_3, x_2) = -i_S$$

$$\left.\begin{aligned} v_L &= L\, di_L/dt \\ &= V_{dd} - R_L i_L - (v_1 + v_2) \end{aligned}\right\} \Rightarrow L\frac{dx_1}{dt} = V_{dd} - R_L x_1 - x_2 - x_3$$

$$i_L = i_D + C_1 \frac{dv_1}{dt} \quad \Longrightarrow \quad C_1 \frac{dx_2}{dt} = x_1 - f(u - x_3, x_2)$$

$$C_2 \frac{dv_2}{dt} = C_1 \frac{dv_1}{dt} - \left(i_S + \frac{1}{R_s}[v_2 - V_{ss}]\right)$$

$$\Longrightarrow \quad C_2 \frac{dx_3}{dt} = [x_1 - f(u - x_3, x_2)] + f(u - x_3, x_2) - \frac{1}{R_s} x_3 + \frac{V_{ss}}{R_s}$$

also $y = v_D = v_1 + v_2 = x_2 + x_3$

a)
$$\Longrightarrow \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\frac{R_L}{L} & -\frac{1}{L} & -\frac{1}{L} \\ \frac{1}{C_1} & 0 & 0 \\ \frac{1}{C_2} & 0 & -\frac{1}{C_2 R_s} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} V_{dd}/L \\ -f(u - x_3, x_2)/C_1 \\ V_{ss}/(R_s C_2) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

b) If $i_D = g_m v_{GS}$ then the equations are linear affine and all solutions are exponential with possibly time multipliers; these are not chaotic, so no chaos will result when $i_D = g_m v_{GS}$ (the solutions can be "almost periodic" which may look a bit like chaotic ones).

3. II. Synthesis with op-amps.

a) $V_3 = V_4 = G(s)\cdot(V_+ - V_-)$ (1)

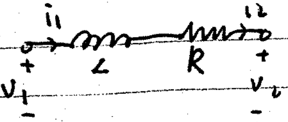
$V_+ - V_- = \frac{R_1}{R_1+R_2}(V_1 - V_2)$ (2)

$G(s) = \frac{G_0}{as+1}$ (5)

$$\left\{\begin{array}{l} V_1 = V_2 + \frac{R_1+R_2}{R_1\cdot G_s} V_4 \quad (3) \\ V_3 = V_4 \quad (4) \end{array}\right.$$

$$\therefore \boxed{A_1 = \begin{bmatrix} 1 & \frac{R_1+R_2}{R_1\cdot G_0}\cdot(as+1) \\ 0 & 1 \end{bmatrix}} \quad (6)$$

b)



$$\begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} 1 & Ls+R \\ 0 & 1 \end{bmatrix}\begin{bmatrix} V_2 \\ i_2 \end{bmatrix} \quad (7)$$

$$\boxed{\left\{\begin{array}{ll} L = \frac{a(R_1+R_2)}{R_1 G_0} & (8) \\ R = \frac{R_1+R_2}{R_1 G_0} & (9) \end{array}\right.}$$

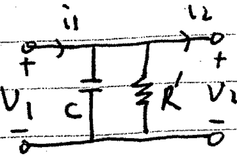
c) $V_1 = V_2 = G(s)\cdot(V_+ - V_-)$ (10)

$V_+ - V_- = \frac{R_1}{R_1+R_2}(V_3 - V_4)$ (11)

$$\left\{\begin{array}{ll} V_1 = V_2 & (12) \\ V_3 = \frac{R_1+R_2}{R_1 G(s)} V_2 + V_4 & (13) \end{array}\right.$$

$$\boxed{A_2 = \begin{bmatrix} 1 & 0 \\ \frac{R_1+R_2}{R_1 G_0}(as+1) & 1 \end{bmatrix}} \quad (14)$$

d)



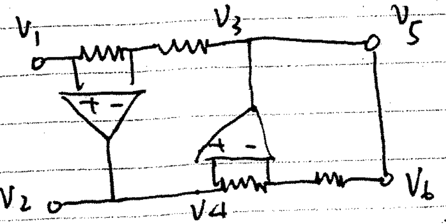
$V_1 = V_2 = (i_1 - i_2)\cdot(R' \| \frac{1}{Cs})$ (15)

$R' \| \frac{1}{Cs} = \frac{R'}{R'Cs+1}$ (16)

$$\begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Cs + \frac{1}{R'} & 1 \end{bmatrix}\begin{bmatrix} V_2 \\ i_2 \end{bmatrix} \quad (17$$

$$\boxed{\therefore \left\{\begin{array}{ll} C = \frac{a(R_1+R_2)}{R_1 G_0} & (18) \\ R' = \frac{R_1 G_0}{R_1+R_2} & (19) \end{array}\right.}$$

e)



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = A_1\cdot\begin{bmatrix} V_3 \\ V_4 \end{bmatrix} \quad (20) \qquad \begin{bmatrix} V_3 \\ V_4 \end{bmatrix} = A_2\cdot\begin{bmatrix} V_5 \\ V_6 \end{bmatrix} \quad (21)$$

$$\therefore \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = A_1\cdot A_2\begin{bmatrix} V_5 \\ V_6 \end{bmatrix} \quad (22)$$

$$A_1\cdot A_2 = \begin{bmatrix} 1+\left(\frac{R_1+R_2}{R_1 G_0}\right)^2(as+1)^2 & \frac{R_1+R_2}{R_1 G_0}(as+1) \\ \frac{R_1+R_2}{R_1 G_0}(as+1) & 1 \end{bmatrix} \quad (23)$$

$A = \frac{R_1+R_2}{R_1}\cdot\frac{1}{G_0}$ 2

$$\boxed{T(s) = \frac{Aas + (A+1)}{A^2a^2s^2 + (2A^2+A)as + (A^2+A}} \quad (26)$$

$V_5 = V_6$, $V_{in} = V_1$ $V_2 = V_{out}$

$$T(s) = \frac{V_{out}}{V_{in}} = \frac{\frac{(R_1+R_2)a}{R_1 G_0}\cdot s + \left(\frac{R_1+R_2}{R_1 G_0} + 1\right)}{\left[\frac{(R_1+R_2)a}{R_1 G_0}\right]^2 s^2 + 2\left[\frac{(R_1+R_2)^2 a}{R_1^2 G_0^2}\right]s + \left[\left(\frac{R_2+R_1}{R_1 G_0}\right)^2 + \frac{R_1+R_2}{R_1 G_0} + 1\right]} \quad (24)$$