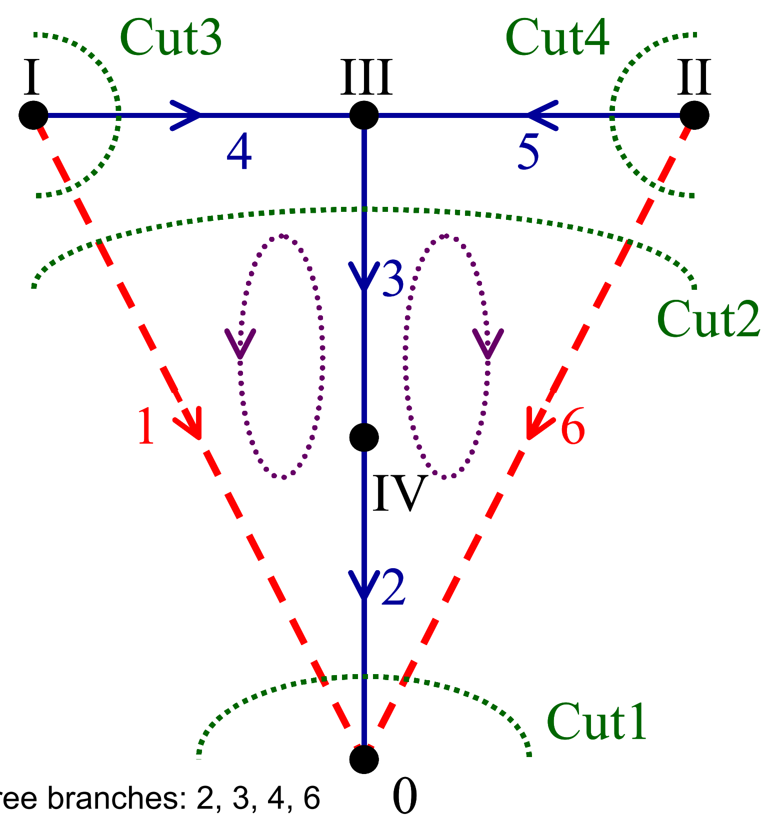
file: c:\jiangyu\Courses_T\enee610\Fall03\HomeWork\HW5_2.mcd
HW5: Problem2 solution JY 10/20/03



Tree branches: 2, 3, 4, 6

Cut set 1: (1, 2, 6) i1+i2+i6=0
Cut set 2: (1, 3, 6) i1+i3+i6=0
Cut set 3: (1, 4) i1+i4=0
Cut set 4: (5, 6) i5+i6=0

Cut set Matrix:

$$C := \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Tie set 1: (1, 2, 3, 4) V1-V2-V3-V4=0
Tie set 2: (2, 3, 5, 6) -V2-V3-V5+V6=0

Tie set Matrix:

$$T := \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 \\ 0 & -1 & -1 & 0 & -1 & 1 \end{bmatrix}$$

a) (10)

State Vector: $X := \begin{bmatrix} V3 \\ i2 \\ V6 \end{bmatrix}$

KCL: $i2 - i3 := 0 \qquad i3 := i2$

$i3 := C \cdot s \cdot V3$

Equation 1): $s \cdot V3 := \frac{1}{C} \cdot i2$

KVL: $-V2 - V3 - V5 + V6 := 0 \qquad V2 := -V3 - V5 + V6$

$$\begin{bmatrix} V4 \\ V5 \end{bmatrix} := \begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix} \cdot \begin{bmatrix} i4 \\ i5 \end{bmatrix} \qquad \begin{bmatrix} V4 \\ V5 \end{bmatrix} := \begin{bmatrix} r \cdot i5 \\ -r \cdot i4 \end{bmatrix}$$

$V2 := V3 - r \cdot i4 - V6$

KCL: $i1 + i4 := 0 \quad i4 := -i1 \qquad i1 := -I1 \qquad i4 := I1$

$V2 := -V3 + V6 + r \cdot I1$

$V2 := L \cdot s \cdot i2$

Equation 2): $s \cdot i2 := \left(\frac{-1}{L} \cdot V3 + \frac{1}{L} \cdot V6 \right) + \frac{r}{L} \cdot I1$

KCL: $i1 + i2 + i6 := 0 \qquad i1 := -I1 \qquad i6 := \frac{V6}{R}$

Equation 3): $R \cdot i2 + V6 - R \cdot I1 := 0$

$$s \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} V3 \\ i2 \\ V6 \end{bmatrix} := \begin{bmatrix} 0 & \frac{1}{C} & 0 \\ \frac{-1}{L} & 0 & \frac{1}{L} \\ 0 & R & 1 \end{bmatrix} \cdot \begin{bmatrix} V3 \\ i2 \\ V6 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{r}{L} \\ -R \end{bmatrix} \cdot I1$$

Semistate Equations:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \frac{d}{dt} \begin{bmatrix} V3 \\ i2 \\ V6 \end{bmatrix} := \begin{bmatrix} 0 & \frac{1}{C} & 0 \\ \frac{-1}{L} & 0 & \frac{1}{L} \\ 0 & R & 1 \end{bmatrix} \cdot \begin{bmatrix} V3 \\ i2 \\ V6 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{r}{L} \\ -R \end{bmatrix} \cdot I1$$

$y := (0 \;\; 0 \;\; 1) \cdot X$

b) (15)

State Vector: $X := \begin{bmatrix} V3 \\ i2 \\ V1 \end{bmatrix}$

KVL: $V1 - V4 + V5 - V6 := 0$

$$\begin{bmatrix} V4 \\ V5 \end{bmatrix} := \begin{bmatrix} r \cdot i5 \\ -r \cdot i4 \end{bmatrix} \qquad V6 := V1 - r \cdot (i4 + i5)$$

KCL: $i4 + i5 - i3 := 0 \quad i4 + i5 := i2$

$$i2 - i3 := 0$$

$$V6 := V1 - r \cdot i2$$

Plug V6 in equation 2) and 3):

Equation 1): stays the same: $s \cdot V3 := \frac{1}{C} \cdot i2$

Equation 4) (from Equation 2)):

$$s \cdot i2 := \left(\frac{-1}{L} \cdot V3 - \frac{r}{L} \cdot i2 + \frac{1}{L} \cdot V1 \right) + \frac{r}{L} \cdot I1$$

Equation 5) (from Equation 3)): $(R - r) \cdot i2 + V1 - R \cdot I1 := 0$

$$s \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} V3 \\ i2 \\ V1 \end{bmatrix} := \begin{bmatrix} 0 & \frac{1}{C} & 0 \\ \frac{-1}{L} & \frac{-r}{L} & \frac{1}{L} \\ 0 & R - r & 1 \end{bmatrix} \cdot \begin{bmatrix} V3 \\ i2 \\ V1 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{r}{L} \\ -R \end{bmatrix} \cdot I1$$

Semistate Equations:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \frac{d}{dt} \begin{bmatrix} V3 \\ i2 \\ V1 \end{bmatrix} := \begin{bmatrix} 0 & \frac{1}{C} & 0 \\ \frac{-1}{L} & \frac{-r}{L} & \frac{1}{L} \\ 0 & R - r & 1 \end{bmatrix} \cdot \begin{bmatrix} V3 \\ i2 \\ V1 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{r}{L} \\ -R \end{bmatrix} \cdot I1$$

$$y := \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \cdot X$$

$$\begin{bmatrix} s & \frac{-1}{C} & 0 \\ \frac{1}{L} & s+\frac{r}{L} & \frac{-1}{L} \\ 0 & r-R & -1 \end{bmatrix} \cdot \begin{bmatrix} V3 \\ i2 \\ V1 \end{bmatrix} := \begin{bmatrix} 0 \\ \frac{r \cdot I1}{L} \\ -R \cdot I1 \end{bmatrix}$$

$$\begin{bmatrix} V3 \\ i2 \\ V1 \end{bmatrix} := \begin{bmatrix} \frac{(s\cdot L+R)\cdot C}{s^2\cdot C\cdot L+s\cdot C\cdot R+1} & \frac{L}{s^2\cdot C\cdot L+s\cdot C\cdot R+1} & \frac{-1}{s^2\cdot C\cdot L+s\cdot C\cdot R+1} \\ \frac{-C}{s^2\cdot C\cdot L+s\cdot C\cdot R+1} & \frac{s\cdot L\cdot C}{s^2\cdot C\cdot L+s\cdot C\cdot R+1} & \frac{-s\cdot C}{s^2\cdot C\cdot L+s\cdot C\cdot R+1} \\ \frac{(R-r)\cdot C}{s^2\cdot C\cdot L+s\cdot C\cdot R+1} & \frac{-s\cdot (R-r)\cdot L\cdot C}{s^2\cdot C\cdot L+s\cdot C\cdot R+1} & \frac{-\left(s^2\cdot C\cdot L+s\cdot C\cdot r+1\right)}{s^2\cdot C\cdot L+s\cdot C\cdot R+1} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \frac{r\cdot I1}{L} \\ -R\cdot I1 \end{bmatrix}$$

$$V1 := \frac{R\cdot L\cdot C\cdot s^2 + r^2\cdot C\cdot s + R}{s^2\cdot C\cdot L + s\cdot C\cdot R + 1}\cdot I1 \qquad Zin := \frac{V1}{I1}$$

Input impedance:

$$Zin := \frac{R\cdot L\cdot C\cdot s^2 + r^2\cdot C\cdot s + R}{s^2\cdot C\cdot L + s\cdot C\cdot R + 1}$$

Equation 1): $s\cdot V3 := \frac{1}{C}\cdot i2$

Equation 5): $(R-r)\cdot i2 + V1 - R\cdot I1 := 0$

$V1 := -(R-r)\cdot i2 + R\cdot I1$ Plug in Equation 4.

Equation 6) (from Equation 4))

$$s\cdot i2 := \left(\frac{-1}{L}\cdot V3 - \frac{R}{L}\cdot i2\right) + \frac{R+r}{L}\cdot I1$$

State Equations:

$$\frac{d}{dt}\begin{bmatrix} V3 \\ i2 \end{bmatrix} := \begin{bmatrix} 0 & \frac{1}{C} \\ \frac{-1}{L} & \frac{-R}{L} \end{bmatrix} \cdot \begin{bmatrix} V3 \\ i2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{R+r}{L} \end{bmatrix} \cdot I1$$

c) and d) (15)

$$\begin{bmatrix} x1 \\ x2 \end{bmatrix} := \begin{bmatrix} V3 \\ i2 \end{bmatrix} \qquad r := 2 \qquad L := 2$$
$$C := 2 \qquad R := 2$$

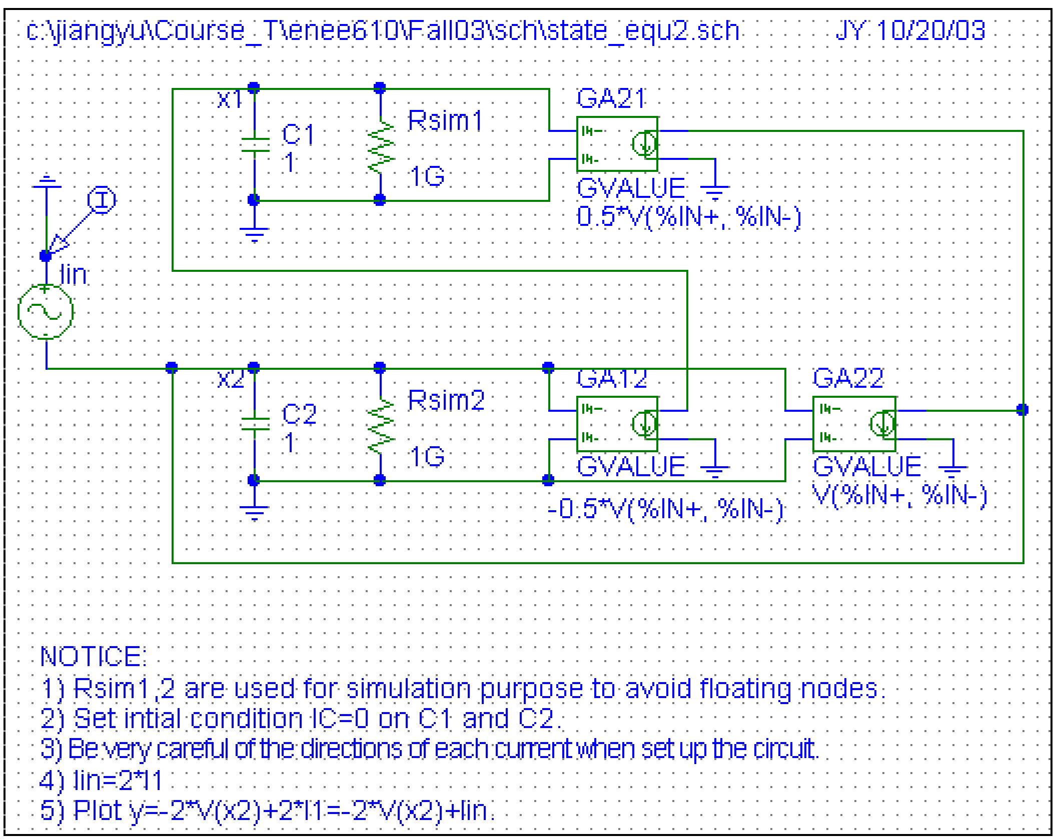
Output VII(t)=V6=-R*i2+R*I1

$$\frac{d}{dt}\begin{bmatrix} x1 \\ x2 \end{bmatrix} := \begin{bmatrix} 0 & 0.5 \\ -0.5 & -1 \end{bmatrix} \cdot \begin{bmatrix} x1 \\ x2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} \cdot I1$$

$$\frac{dx1}{dt} := 0.5 \cdot x2$$

$$\frac{dx2}{dt} := -0.5 \cdot x1 - x2 + 2 \cdot I1$$

$$y := (0 \quad -2) \cdot X + 2 \cdot u$$



Simulation results: