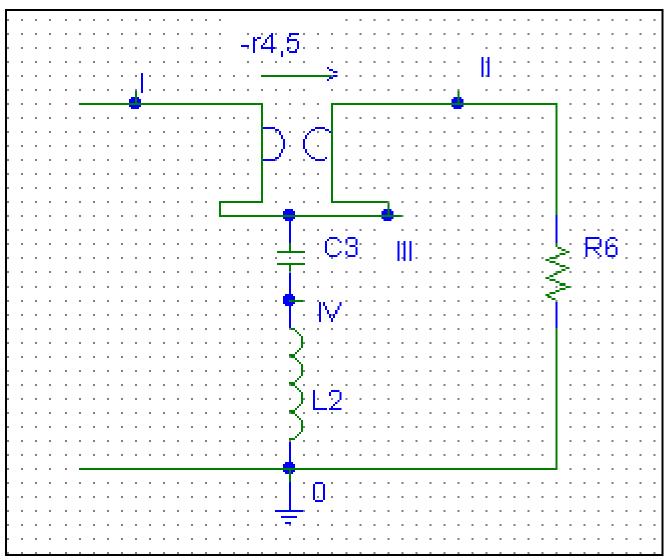
file: c:\jiangyu\Courses_T\enee610\Fall03\HomeWork\HW5_1.mcd
HW5: Problem1 solution JY 10/20/03

a) (10)

$$YN := \begin{bmatrix} \frac{1}{s \cdot L} & 0 & 0 & 0 & 0 \\ 0 & s \cdot C & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{r} & 0 \\ 0 & 0 & \frac{1}{r} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{R} \end{bmatrix}$$

$$YNa := \begin{bmatrix} \frac{1}{s \cdot L} & 0 & 0 & 0 & 0 \\ 0 & s \cdot C & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{r} & 0 \\ 0 & 0 & \frac{-1}{r} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{R} \end{bmatrix}$$

Adjoint circuit:

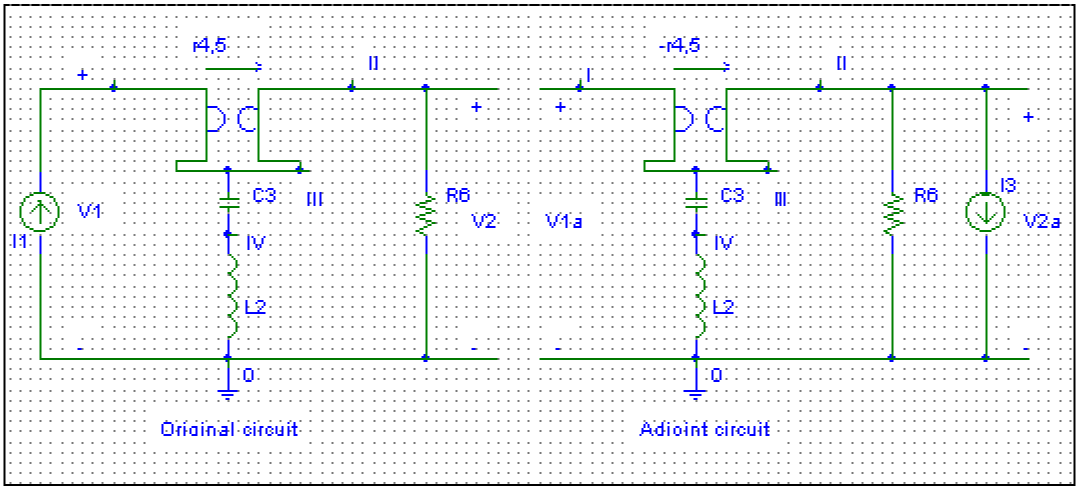


b) (10)

$$T(s) := \frac{VII}{I1}$$

Input: I1
Output: VII (V2)

Terminate original and adjoint circuit:



$V_1*i_1^a+V_2*i_2^a-V_1^a*i_1-V_2^a*i_2=-[i_N^{aT}*V_N-i_N^T*V_N^a]$

Clearly $i_1^a=i_2=0$

So $V_2*i_2^a-V_1^a*i_1=-[i_N^{aT}*V_N-i_N^T*V_N^a]$

Since is i_1 fixed in a, and nothing varies in N^a,

then $di_1/da=0$, $di_2^a/da=0$, and $dV_1^a/da=0$,

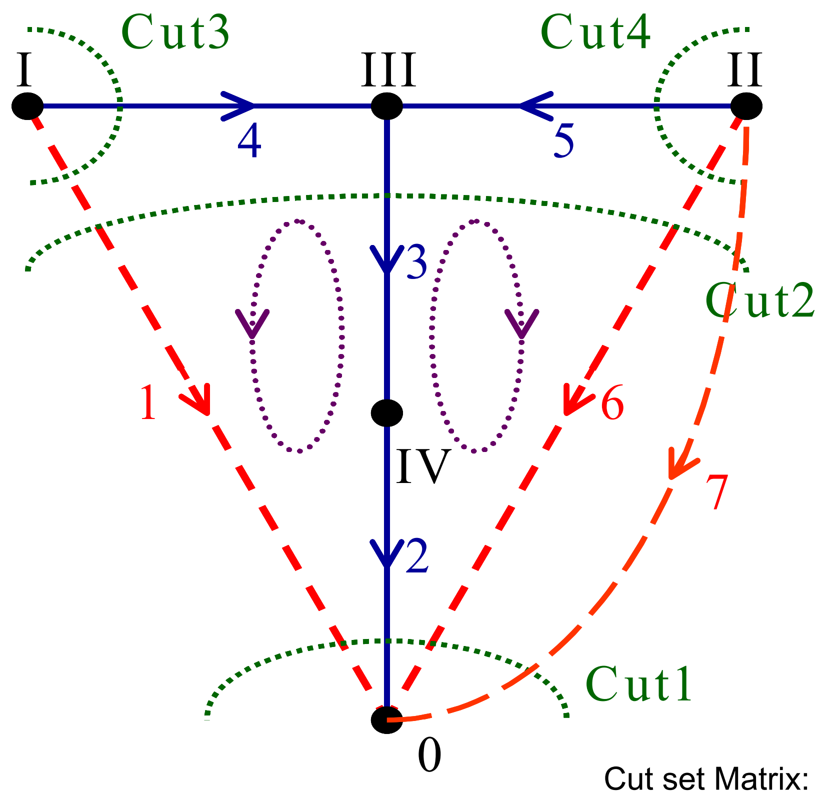
So $i_2^a*dV_2/da=-[i_N^{aT}*V_N-i_N^T*V_N^a]$

Then $dV_2/da=-[i_N^{aT}*V_N-i_N^T*V_N^a]/i_2^a=[V_N^{aT}*(dY_N/da)*V_N]/i_2^a$

$dT(s)/da=dV_2/da=[V_N^{aT}*(dY_N/da)*V_N]/i_2^a$ by setting I1=1

$$I1 := 1 \quad \frac{dT(s)}{da} := \frac{1}{I2a}\cdot\left(VNaT\cdot\frac{dYN}{da}\cdot VN\right)$$

c) (15) Circuit annalysis of the original and adjoint circuit: (10)
They are have the same gragh.



Tree branches: 2, 3, 4, 6

Cut set Matrix:

Cut set 1: (1, 2, 6, 7)	i1+i2+i6+i7=0	
Cut set 2: (1, 3, 6, 7)	i1+i3+i6+i7=0	
Cut set 3: (1, 4)	i1+i4=0	
Cut set 4: (5, 6, 7)	i5+i6+i7=0	

$$C := \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Tie set Matrix:

Tie set 1: (1, 2, 3, 4) V1-V2-V3-V4=0
Tie set 2: (2, 3, 5, 6) -V2-V3-V5+V6=0

$$T := \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & -1 & 1 & 0 \\ 0 & -1 & -1 & 0 & -1 & 0 & 1 \end{bmatrix}$$

Using equality $-C*j=-C*Ybb*C^T*Vt$

$$j := \begin{bmatrix} -I1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ I2a \end{bmatrix}$$

$$Left := -C \cdot j$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} -I1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ I2 \end{bmatrix} = \begin{bmatrix} -I1 + I2a \\ -I1 + I2a \\ -I1 \\ I2a \end{bmatrix}$$

For orinal circuit, I1=1, I2a=0

Admittance matrix:

$$Ybb := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{s\cdot L} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & s\cdot C & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{r} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{r} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{R} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Evaluate CYbbCT=C*Ybb*C^T

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{s\cdot L} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & s\cdot C & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{r} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{r} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{R} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

CYbbCT=

$$\begin{bmatrix} \frac{1}{s\cdot L} + \frac{1}{R} & \frac{1}{R} & 0 & \frac{1}{R} \\ \frac{1}{R} & s\cdot C + \frac{1}{R} & 0 & \frac{1}{R} \\ 0 & 0 & 0 & \frac{-1}{r} \\ \frac{1}{R} & \frac{1}{R} & \frac{1}{r} & \frac{1}{R} \end{bmatrix}$$

-C*j=CYbbCT*Vb

$$\begin{bmatrix} I1 \\ I1 \\ I1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{s\cdot L}+\frac{1}{R} & \frac{1}{R} & 0 & \frac{1}{R} \\ \frac{1}{R} & s\cdot C+\frac{1}{R} & 0 & \frac{1}{R} \\ 0 & 0 & 0 & \frac{-1}{r} \\ \frac{1}{R} & \frac{1}{R} & \frac{1}{r} & \frac{1}{R} \end{bmatrix} x \begin{bmatrix} V2 \\ V3 \\ V4 \\ V5 \end{bmatrix}$$

$$CYbbCT^{-1} = \begin{bmatrix} \frac{L\cdot(s\cdot C\cdot R+1)\cdot s}{s\cdot C\cdot R+1+s^2\cdot L\cdot C} & \frac{-L\cdot s}{s\cdot C\cdot R+1+s^2\cdot L\cdot C} & \frac{r\cdot L\cdot C\cdot s^2}{s\cdot C\cdot R+1+s^2\cdot L\cdot C} & 0 \\ \frac{-L\cdot s}{s\cdot C\cdot R+1+s^2\cdot L\cdot C} & \frac{(R+s\cdot L)}{\left(s\cdot C\cdot R+1+s^2\cdot L\cdot C\right)} & \frac{r}{\left(s\cdot C\cdot R+1+s^2\cdot L\cdot C\right)} & 0 \\ \frac{-r\cdot L\cdot C\cdot s^2}{s\cdot C\cdot R+1+s^2\cdot L\cdot C} & \frac{-r}{\left(s\cdot C\cdot R+1+s^2\cdot L\cdot C\right)} & \frac{r^2\cdot C\cdot s}{s\cdot C\cdot R+1+s^2\cdot L\cdot C} & r \\ 0 & 0 & -r & 0 \end{bmatrix}$$

$$\begin{bmatrix} V2 \\ V3 \\ V4 \\ V5 \end{bmatrix} := CYbbCt^{-1}\cdot\begin{bmatrix} I1 \\ I1 \\ I1 \\ 0 \end{bmatrix} \qquad V6 := V2 + V3 + V5$$

For I1=1:

$$\begin{bmatrix} V2 \\ V3 \\ V4 \\ V5 \end{bmatrix} := CYbbCt^{-1}\cdot\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$V2 := \frac{(R + r)\cdot L\cdot C\cdot s^2}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C}$$

$$V3 := \frac{R + r}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C}$$

$$V4 := \frac{-r\cdot L\cdot C\cdot s^2 - r + r^2\cdot C\cdot s}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C}$$

$$V5 := -r$$

$$V6 := \frac{R\cdot L\cdot C\cdot s^2 + R - R\cdot r\cdot C\cdot s}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C}$$

$$VN := \begin{bmatrix} \dfrac{(R + r)\cdot L\cdot C\cdot s^2}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C} \\ \dfrac{R + r}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C} \\ \dfrac{-r\cdot L\cdot C\cdot s^2 - r + r^2\cdot C\cdot s}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C} \\ -r \\ \dfrac{R\cdot L\cdot C\cdot s^2 + R - R\cdot r\cdot C\cdot s}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C} \end{bmatrix}$$

For adjoint circuit, I1=0

Admittance matrix: $Ybba := Ybb^T$

Evaluate CYbbaCT=C*Ybba*C^T

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{s\cdot L} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & s\cdot C & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{r} & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{r} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{R} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

CYbbaCT=

$$\begin{matrix} \frac{1}{s\cdot L} + \frac{1}{R} & \frac{1}{R} & 0 & \frac{1}{R} \\ \frac{1}{R} & s\cdot C + \frac{1}{R} & 0 & \frac{1}{R} \\ 0 & 0 & 0 & \frac{1}{r} \\ \frac{1}{R} & \frac{1}{R} & \frac{-1}{r} & \frac{1}{R} \end{matrix}$$

Clearly:

$$CYbbaCT := CYbbCT^T$$

-C*j=CYbbaCT*Vb

$$\begin{bmatrix} -I2a \\ -I2a \\ 0 \\ I2a \end{bmatrix} = \begin{bmatrix} \frac{1}{s\cdot L}+\frac{1}{R} & \frac{1}{R} & 0 & \frac{1}{R} \\ \frac{1}{R} & s\cdot C+\frac{1}{R} & 0 & \frac{1}{R} \\ 0 & 0 & 0 & \frac{1}{r} \\ \frac{1}{R} & \frac{1}{R} & \frac{-1}{r} & \frac{1}{R} \end{bmatrix} \times \begin{bmatrix} V2a \\ V3a \\ V4a \\ V5a \end{bmatrix}$$

$$CYbbaCT^{-1} = \begin{bmatrix} \frac{L\cdot(s\cdot C\cdot R+1)\cdot s}{s\cdot C\cdot R+1+s^2\cdot L\cdot C} & \frac{-L\cdot s}{s\cdot C\cdot R+1+s^2\cdot L\cdot C} & \frac{-r\cdot L\cdot C\cdot s^2}{s\cdot C\cdot R+1+s^2\cdot L\cdot C} & 0 \\ \frac{-L\cdot s}{s\cdot C\cdot R+1+s^2\cdot L\cdot C} & \frac{s\cdot L+R}{s\cdot C\cdot R+1+s^2\cdot L\cdot C} & \frac{-r}{s\cdot C\cdot R+1+s^2\cdot L\cdot C} & 0 \\ \frac{r\cdot L\cdot C\cdot s^2}{s\cdot C\cdot R+1+s^2\cdot L\cdot C} & \frac{r}{s\cdot C\cdot R+1+s^2\cdot L\cdot C} & \frac{r^2\cdot C\cdot s}{s\cdot C\cdot R+1+s^2\cdot L\cdot C} & -r \\ 0 & 0 & r & 0 \end{bmatrix}$$

$$\begin{bmatrix} V2a \\ V3a \\ V4a \\ V5a \end{bmatrix} := CYbbaCt^{-1}\cdot\begin{bmatrix} -I2a \\ -I2a \\ 0 \\ -I2a \end{bmatrix}$$

$$V6a := V2a + V3a + V5a$$

$$V2a := \frac{-R\cdot L\cdot C\cdot s^2\cdot I2a}{s\cdot C\cdot R+1+s^2\cdot L\cdot C}$$

$$V5a := 0$$

$$V6a := \frac{-R\cdot(L\cdot C\cdot s^2+1)\cdot I2a}{s\cdot C\cdot R+1+s^2\cdot L\cdot C}$$

$$V3a := \frac{-R\cdot I2a}{s\cdot C\cdot R+1+s^2\cdot L\cdot C}$$

$$V4a := \frac{R\cdot r\cdot C\cdot s\cdot I2a}{s\cdot C\cdot R+1+s^2\cdot L\cdot C}$$

$$YNa := \begin{bmatrix} \dfrac{-R\cdot L\cdot C\cdot s^2\cdot I2a}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C} \\ \dfrac{-R\cdot I2a}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C} \\ \dfrac{R\cdot r\cdot C\cdot s\cdot I2a}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C} \\ 0 \\ \dfrac{-R\cdot(L\cdot C\cdot s^2 + 1)\cdot I2a}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C} \end{bmatrix} \qquad VN := \begin{bmatrix} \dfrac{(R + r)\cdot L\cdot C\cdot s^2}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C} \\ \dfrac{R + r}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C} \\ \dfrac{-r\cdot L\cdot C\cdot s^2 - r + r^2\cdot C\cdot s}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C} \\ -r \\ \dfrac{R\cdot L\cdot C\cdot s^2 + R - R\cdot r\cdot C\cdot s}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C} \end{bmatrix}$$

$$\frac{dT}{da} := \frac{1}{I2a}\cdot\left[VNaT\cdot\left(\frac{dYN}{da}\right)\cdot VN\right]$$

Calculate sensitivity with respect to r: (5) $S(C) := \frac{r}{T}\cdot\frac{dT}{dr}$

$a := r$

$$\frac{dYN}{dr} := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{1}{r^2} & 0 \\ 0 & 0 & \dfrac{-1}{r^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{dT}{dr} := \frac{1}{I2a\cdot r^2}\cdot(V4a\cdot V5 - V5a\cdot V4)$$

$$\frac{dT}{dr} := \frac{-R\cdot C\cdot s}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C}$$

$$S(C) := \frac{-r\cdot C\cdot s}{L\cdot C\cdot s^2 + 1 - r\cdot C\cdot s}$$

d) (5) Calculate sensitivity with respect to C $S(C) := \frac{C}{T}\cdot\frac{dT}{dC}$

$a := C$

$$\frac{dYN}{dC} := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & s & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{dT}{dC} := \frac{1}{I2a}\cdot V3a\cdot s\cdot V3 \qquad \frac{dT}{dC} := \frac{-R\cdot(R + r)\cdot s}{\left(s\cdot C\cdot R + 1 + s^2\cdot L\cdot C\right)^2}$$

$$S(C) := \frac{-(R + r)\cdot C\cdot s}{\left(s\cdot C\cdot R + 1 + s^2\cdot L\cdot C\right)\cdot(L\cdot C\cdot s^2 + 1 - r\cdot C\cdot s)}$$