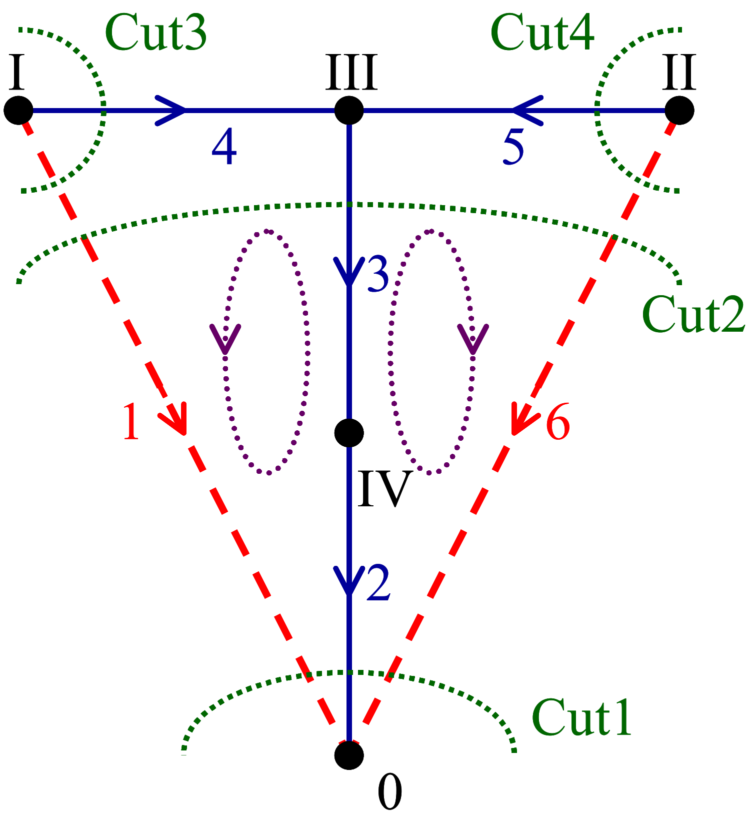
file: c:\jiangyu\Courses_T\enee610\Fall03\HomeWork\HW4_3.mcd
HW4: Problem3 solution JY 10/06/03

a) (10)



b) (15)

Tree branches: 2, 3, 4, 6

Cut set 1: (1, 2, 6) i1+i2+i6=0
Cut set 2: (1, 3, 6) i1+i3+i6=0
Cut set 3: (1, 4) i1+i4=0
Cut set 4: (5, 6) i5+i6=0

Cut set Matrix:

$$C := \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Tie set 1: (1, 2, 3, 4) V1-V2-V3-V4=0
Tie set 2: (2, 3, 5, 6) -V2-V3-V5+V6=0

Tie set Matrix:

$$T := \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 \\ 0 & -1 & -1 & 0 & -1 & 1 \end{bmatrix}$$

Admittance matrix:

$$Ybb := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{s \cdot L} & 0 & 0 & 0 & 0 \\ 0 & 0 & s \cdot C & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{r} & 0 \\ 0 & 0 & 0 & \frac{1}{r} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{R} \end{bmatrix}$$

c) (10)

Using equality -C*j=-C*Ybb*C^T*Vt

$$j := \begin{bmatrix} -is \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Left := -C \cdot j$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} -is \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -is \\ -is \\ -is \\ 0 \end{bmatrix}$$

$$Left := \begin{bmatrix} is \\ is \\ is \\ 0 \end{bmatrix}$$

$$CYbbCT := C \cdot Ybb \cdot C^T$$

$$C = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad Ybb := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{s \cdot L} & 0 & 0 & 0 & 0 \\ 0 & 0 & s \cdot C & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{r} & 0 \\ 0 & 0 & 0 & \frac{1}{r} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{R} \end{bmatrix} \quad C^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

Evaluate C*Ybb

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{s \cdot L} & 0 & 0 & 0 & 0 \\ 0 & 0 & s \cdot C & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{r} & 0 \\ 0 & 0 & 0 & \frac{1}{r} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{R} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{s \cdot L} & 0 & 0 & 0 & \frac{1}{R} \\ 0 & 0 & s \cdot C & 0 & 0 & \frac{1}{R} \\ 0 & 0 & 0 & 0 & \frac{-1}{r} & 0 \\ 0 & 0 & 0 & \frac{1}{r} & 0 & \frac{1}{R} \end{bmatrix}$$

Evaluate CYbbCT=(C*Ybb)*C^T

$$\text{CYbbCT} = \begin{bmatrix} 0 & \frac{1}{s \cdot L} & 0 & 0 & 0 & \frac{1}{R} \\ 0 & 0 & s \cdot C & 0 & 0 & \frac{1}{R} \\ 0 & 0 & 0 & 0 & \frac{-1}{r} & 0 \\ 0 & 0 & 0 & \frac{1}{r} & 0 & \frac{1}{R} \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{s \cdot L} + \frac{1}{R} & \frac{1}{R} & 0 & \frac{1}{R} \\ \frac{1}{R} & s \cdot C + \frac{1}{R} & 0 & \frac{1}{R} \\ 0 & 0 & 0 & \frac{-1}{r} \\ \frac{1}{R} & \frac{1}{R} & \frac{1}{r} & \frac{1}{R} \end{bmatrix}$$

-C*j=CYbbCT*Vb

$$\begin{bmatrix} is \\ is \\ is \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{s \cdot L} + \frac{1}{R} & \frac{1}{R} & 0 & \frac{1}{R} \\ \frac{1}{R} & s \cdot C + \frac{1}{R} & 0 & \frac{1}{R} \\ 0 & 0 & 0 & \frac{-1}{r} \\ \frac{1}{R} & \frac{1}{R} & \frac{1}{r} & \frac{1}{R} \end{bmatrix} \times \begin{bmatrix} V2 \\ V3 \\ V4 \\ V5 \end{bmatrix}$$

$$\det(CYbbCT) = \frac{1}{R}\cdot\frac{\left(s\cdot C\cdot R + 1 + s^2\cdot L\cdot C\right)}{\left(s\cdot L\cdot r^2\right)}$$

$$CYbbCT^{-1} =$$

$$\begin{bmatrix} \frac{L\cdot(s\cdot C\cdot R+1)\cdot s}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C} & \frac{-L\cdot s}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C} & \frac{r\cdot L\cdot C\cdot s^2}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C} & 0 \\ \frac{-L\cdot s}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C} & \frac{(R + s\cdot L)}{\left(s\cdot C\cdot R + 1 + s^2\cdot L\cdot C\right)} & \frac{r}{\left(s\cdot C\cdot R + 1 + s^2\cdot L\cdot C\right)} & 0 \\ \frac{-r\cdot L\cdot C\cdot s^2}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C} & \frac{-r}{\left(s\cdot C\cdot R + 1 + s^2\cdot L\cdot C\right)} & \frac{r^2\cdot C\cdot s}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C} & r \\ 0 & 0 & -r & 0 \end{bmatrix}$$

$$\begin{bmatrix} V2 \\ V3 \\ V4 \\ V5 \end{bmatrix} := CYbbCt^{-1}\cdot\begin{bmatrix} is \\ is \\ is \\ 0 \end{bmatrix}$$

$$Vin := V1 \qquad V1 := V2 + V3 + V4$$

$$Iin := is \qquad Zin := \frac{Vin}{Iin}$$

$$V2 := \frac{(R + r)\cdot L\cdot C\cdot s^2}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C}\cdot is \qquad V3 := \frac{R + r}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C}\cdot is$$

$$V4 := \frac{-r\cdot L\cdot C\cdot s^2 - r + r^2\cdot C\cdot s}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C}\cdot is \qquad V5 := -r\cdot is$$

$$V1 := \frac{R\cdot L\cdot C\cdot s^2 + R + r^2\cdot C\cdot s}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C}\cdot is$$

Input impedance Zin:

$$Zin := \frac{R\cdot L\cdot C\cdot s^2 + R + r^2\cdot C\cdot s}{s\cdot C\cdot R + 1 + s^2\cdot L\cdot C}$$