ENEE 610

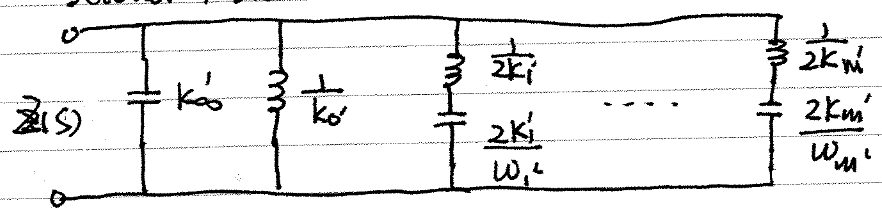
HW 4

Problem 1:

Lossless Passive finite Circuit:

$Y(S)$: $Y(S) = k_\infty' S + \frac{k_0'}{S} + \sum_{i=1}^{m} \frac{2k_i' S}{S^2 + \omega_i^2}$ (1)

Second Foster realization: (LC)



(10) a) $Y(S) = -Y(-S)$ (2)

$SY(S) = k_\infty' S^2 + k_0' + \sum_{i=1}^{m} \frac{2k_i' S^2}{S^2 + \omega_i^2}$ (3)

$y(s) = k_\infty' S + k_0' + \sum_{i=1}^{m} \frac{2k_i' S}{S + \sigma_i}$ (4) with $\sigma_i = \omega_i^2$ (5)

clearly, $y(s^2) = s\,Y(S)$ (6)

$\therefore$ $S\,Y(s)$ is a <u>rational</u> function $y(.)$ of s^2, $y(s^2) = s\,Y(s)$

(15) b) prove $y(s)$ is positive real.

I. Using Definition

① $y(s) = k_\infty' S + k_0' + \sum_{i=1}^{m} \frac{2k_i' S}{S + \sigma_i}$ is real when S is real.

$s = \sigma + j\omega$ (7)

② $y(s) = k_\infty'(\sigma + j\omega) + k_0' + \sum_{i=1}^{m} \frac{2k_i'(\sigma + j\omega)}{(\sigma + \sigma_i) + j\omega}$

$= k_\infty'(\sigma + j\omega) + k_0' + \sum_{i=1}^{m} \frac{2k_i'[\sigma(\sigma + \sigma_i) + \omega^2 + \sigma_i j\omega]}{(\sigma + \sigma_i)^2 + \omega^2}$ (8)

$\therefore Re[y(s)] = k_\infty' \sigma + k_0' + \sum_{i=1}^{m} 2k_i'[\sigma(\sigma + \sigma_i) + \omega^2] / [(\sigma + \sigma_i)^2 + \omega^2]$ (9)

$Re[y(s)] \geq 0$ when $\sigma \geq 0$

$\therefore$ by definition of positive real

$y(s)$ is positive real.

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Problem 2: b) (Continued)

II. Using Theorem 8.2-2

$$y(s) = k_\infty' s + k_0' + \sum_{i=1}^{m} \frac{2k_i' s}{s+\sigma_i} \qquad (10)$$

① $y(s)$ has no poles in right half of s plane, $\sigma_i > 0$

② $y(s)$ has no poles at $j\omega$ axis

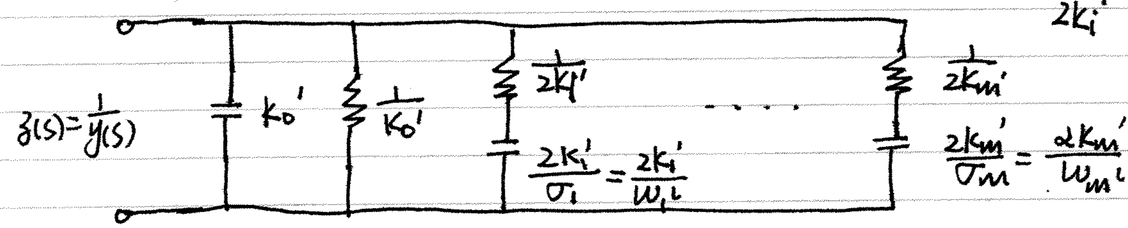
③ $y(j\omega) = k_\infty'(j\omega) + k_0' + \sum_{i=1}^{m} \frac{2k_i' j\omega}{j\omega + \sigma_i}$

$$= k_\infty'(j\omega) + k_0' + \sum_{i=1}^{m} \frac{2k_i' [\omega^2 + j\sigma_i \omega]}{\sigma_i^2 + \omega^2} \qquad (11)$$

$$R[y(j\omega)] = k_0' + \sum_{i=1}^{m} 2k_i' \omega^2 / [\sigma_i^2 + \omega^2] \qquad (12)$$

$\therefore R[y(j\omega)] \geq 0$ for all $0 \leq \omega \leq \infty$

$\therefore y(s)$ is positive real $\quad y(s) = k_\infty' s + k_0' + \sum_{i=1}^{m} \frac{1}{\frac{1}{2k_i'} + \frac{1}{\frac{2k_i'}{\sigma_i} s}} \qquad (13)$



RC realization

Comparing with $Y(s)$ realization: replacing L by R with the same value.

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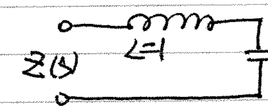
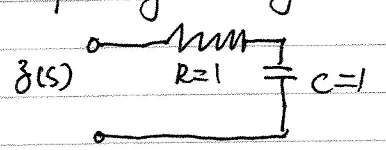
Problem 2: ⑤

a) example: $Y(s)=\frac{s}{s^2+1}$ (1) $\quad Z(s)= s+\frac{1}{s}$ (2)

Foster realization



By replacing L=1 by R=1



$Z(s)=1+\frac{1}{s}$ (3) $\quad Y(s)=\frac{s}{s+1}$ (4)

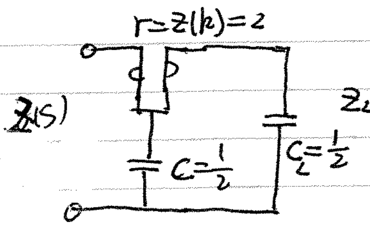
Clearly $\quad Y(s^2)=\frac{s^2}{s^2+1}=s\cdot Y(s)$ (5)

If Richard's function is used.

$Z(s)=-Z(-s)$ (6) $\quad Z(s)+Z(-s)=0$ (7) $\quad k$ can be any value

Choose $k=1$ $\quad Z(s)\big|_{s=1}=2$ (8)

$$Rich(s)=\frac{kZ(s)-sZ(k)}{kZ(k)-sZ(s)}=\frac{s+\frac{1}{s}-2s}{2-s\cdot(s+\frac{1}{s})}=\frac{1-s^2}{(1-s^2)\cdot s}=\frac{1}{s} \quad (9)$$

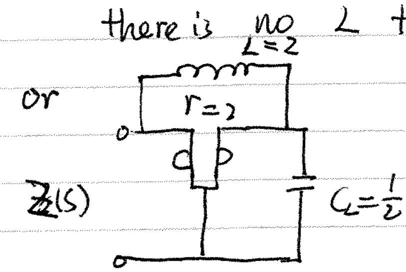


$Z_L=r\cdot Rich(s)=\frac{1}{\frac{1}{2}s}$

$r=Z(k)=2$ (10)
$C=1/kZ(k)=\frac{1}{2}$ (11)
$Z_L=r\cdot Rich(s)=\frac{2}{s}$ (12)

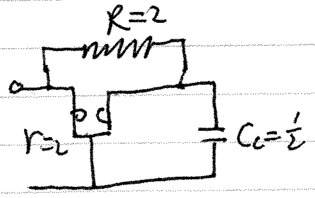
there is no L to be replaced.

or



$r=Z(k)=2$ (13)
$L=\frac{Z(k)}{k}=2$ (14)
$Z_L=r\cdot Rich(s)=\frac{2}{s}$ (15)

replace L=2 by R=2



$$Y=\begin{bmatrix}\frac{1}{R} & -\frac{1}{R}-\frac{1}{r}\\ -\frac{1}{R}+\frac{1}{r} & \frac{1}{R}\end{bmatrix} \quad (16) \qquad Z=\begin{bmatrix}\frac{r^2}{R} & \frac{r^2}{R}+r\\ \frac{r^2}{R}-r & \frac{r^2}{R}\end{bmatrix}=\begin{bmatrix}2 & 4\\ 0 & 2\end{bmatrix} \quad (17)$$

$$Z(s)=Z_{in}=\frac{Z_{11}Z_L+\Delta Z}{Z_L+Z_{22}}=\frac{2\cdot\frac{2}{s}+4}{\frac{2}{s}+2}=2 \quad (18) \qquad Y(s)=\frac{1}{2} \quad (19)$$

Clearly $Y(s^2)\neq s\cdot Y(s)$ (20)

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The reason for none-synthesizable by Degree one real Richards's section.

zeros of even part of $Z_{in}(s)$ are complex number, not real number.

example: $Z(s) = \frac{(s+1)(s+2)}{(s+3)(s+4)}$ (1)

$Z(s)$ is positive real (checked in text book Example 8.2-1 (6), P341)

$$\text{even}\left(Z(s)\right) = \frac{1}{2}\left[\frac{(s+1)(s+2)}{(s+3)(s+4)} + \frac{(-s+1)(-s+2)}{(-s+3)(-s+4)}\right] = \frac{N_v(s)}{D_m(s)} \quad (2)$$

$$N_v(s) = \frac{1}{2}\left[(s^2+3s+2)(s^2-7s+12) + (s^2-3s+2)(s^2+7s+12)\right] \quad (3)$$

$$N_v(s) = s^4 - 7s^2 + 24 \quad (4)$$

$$N_v(s) = 0 \qquad s^2 = \frac{1}{2}\cdot\left(7 \pm j\sqrt{47}\right) \quad (5)$$

$$s_{1,2,3,4} = \pm\sqrt{\frac{7 \pm j\sqrt{47}}{2}} \quad (6)$$ zeros are not real numbers

since
$$\begin{cases} r = Z(k) & (7) \\ c = \frac{1}{kZ(k)} & (8) \\ Z_L = Z(k)\ \mathrm{Rich}(s) & (9) \end{cases}$$
r & c are not real numbers

∴ not synthesizable by degree one real Richards' section