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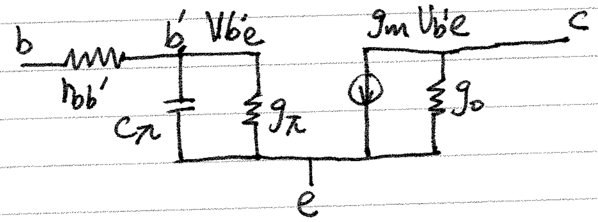
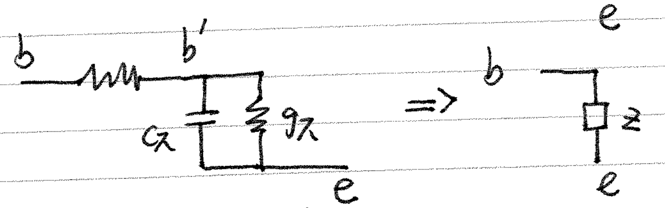
HW 3

Problem 1 a)





$$z = r_{bb'} + \frac{1}{C_\pi S + g_\pi} \qquad (1)$$

$$V_{b'e} = \frac{\frac{1}{C_\pi s + g_\pi}}{r_{bb'} + \frac{1}{C_\pi S + g_\pi}} V_{be} = \frac{V_{be}}{r_{bb'}(C_\pi S + g_\pi) + 1} \qquad (2)$$

$$V_1 = V_{be}, \quad V_2 = V_{ce}, \quad i_1 = i_b, \quad i_2 = i_c \qquad (3)$$

$$i_1 = i_b = \frac{V_{be}}{z} = \frac{C_\pi S + g_\pi}{r_{bb'}(C_\pi S + g_\pi) + 1} V_1 \qquad (4)$$

$$i_2 = i_c = g_m V_{b'e} + g_o V_{CE} = \frac{g_m}{r_{bb'}(C_\pi S + g_\pi) + 1} V_{be} + g_o V_{CE}$$

$$= \frac{g_m}{r_{bb'}(C_\pi S + g_\pi) + 1} V_1 + g_o V_2 \qquad (5)$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \frac{C_\pi S + g_\pi}{r_{bb'}(C_\pi S + g_\pi) + 1} & 0 \\ \frac{g_m}{r_{bb'}(C_\pi S + g_\pi) + 1} & g_o \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad (6)$$

$$Y = \begin{bmatrix} \frac{C_\pi S + g_\pi}{r_{bb'}(C_\pi S + g_\pi) + 1} & 0 \\ \frac{g_m}{r_{bb'}(C_\pi S + g_\pi) + 1} & g_o \end{bmatrix} = \begin{bmatrix} \frac{C_\pi S + g_\pi}{r_{bb'} C_\pi S + (r_{bb'} g_\pi + 1)} & 0 \\ \frac{g_m}{r_{bb'} C_\pi S + (r_{bb'} g_\pi + 1)} & g_o \end{bmatrix} \qquad (7)$$

$$= \begin{bmatrix} \frac{S + \frac{g_\pi}{C_\pi}}{r_{bb'}\left(S + \frac{r_{bb'} g_\pi + 1}{r_{bb'} C_\pi}\right)} & 0 \\ \frac{g_m}{r_{bb'} C_\pi} \cdot \frac{1}{\left(S + \frac{r_{bb'} g_\pi + 1}{\ldots}\right)} & g_o \end{bmatrix}$$

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Problem 1 b)

$$V_T = 26\,mV,\quad I_c = 2mA,\quad V_A = 100\,V,\quad \beta = 50,\quad r_{bb'} = 10\,\Omega,\quad C_\pi = 20pF \qquad (1)$$

$$g_m = \frac{I_c}{V_T} = \frac{2mA}{26mV} = 7.69\times10^{-2}\,\Omega^{-1} \qquad (2)$$

$$g_o = \frac{I_c}{V_A} = \frac{2mA}{100\,V} = 2\times10^{-5}\,\Omega^{-1} \qquad (3)$$

$$g_\pi = \frac{g_m}{\beta} = \frac{7.69\times10^{-2}\,\Omega^{-1}}{50} = 1.538\times10^{-3}\,\Omega^{-1} \qquad (4)$$

$$\frac{g_\pi}{C_\pi} = \frac{1.538\times10^{-3}\,\Omega^{-1}}{20\times10^{-12}\,F} = 7.69\times10^{7}\,(\Omega F)^{-1} \qquad (5)$$

$$\frac{r_{bb'}g_\pi + 1}{r_{bb'}C_\pi} = \frac{10\,\Omega\cdot1.538\times10^{-3}\,\Omega^{-1} + 1}{10\,\Omega\cdot20\times10^{-12}\,F} = 5.077\times10^{9}\,(\Omega F)^{-1} \qquad (6)$$

$$\frac{g_m}{r_{bb'}C_\pi} = \frac{7.69\times10^{-2}\,\Omega^{-1}}{10\,\Omega\cdot20\times10^{-12}\,F} = 3.85\times10^{10}\,\frac{1}{\Omega^2 F} \qquad (7)$$

$$\therefore\ Y = \begin{bmatrix} \dfrac{s + 7.69\times10^{+7}\,(\Omega F)^{-1}}{10\,\Omega\,(s + 5.077\times10^{9}\,\Omega^{-1}F^{-1})} & 0 \\ 3.85\times10^{10}\,\dfrac{1}{\Omega^2 F}\cdot\dfrac{1}{(s + 5.077\times10^{9}\,\Omega^{-1}F^{-1})} & 2\times10^{-5}\,\Omega^{-1} \end{bmatrix}$$

d) Transistor — Passive

small signal hybrid-π equivalent circuit not passive

since not all connections and bias are shown in the model.

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Problem 1 c)

$$H(Y) = \frac{1}{2}\left(Y + Y^{*T}\right) \qquad R(Y) = \frac{1}{2}\left(Y + Y^{*}\right) \quad (1)$$

$$Y = \begin{bmatrix} \frac{1}{A} \cdot \frac{(s+a)}{(s+b)} & 0 \\ B \cdot \frac{1}{(s+b)} & g_0 \end{bmatrix} \quad (2)$$

with $A = 10\Omega$, $B = 3.85\times10^{10} \frac{1}{\Omega^2 F}$, $a = 7.69\times10^{7} (\Omega F)^{-1}$, $b = 5.077\times10^{9} (\Omega F)^{-1}$

and $g_0 = 2\times10^{-5}\ \Omega^{-1}$ (3)

$$H(Y)_{11} = \frac{1}{2}\left(Y_{11} + Y_{11}^{*}\right) = \frac{1}{2A}\cdot\left(\frac{s+a}{s+b} + \frac{s^*+a}{s^*+b}\right) = \frac{1}{A}\cdot\frac{|s|^2 + (a+b)\,Re(s) + ab}{|s|^2 + 2b\,Re(s) + b^2} \quad (4)$$

$$H(Y)_{12} = \frac{1}{2}\left(Y_{12} + Y_{21}^{*}\right) = \frac{B}{2}\cdot\frac{1}{s^*+b} \quad (5)$$

$$H(Y)_{21} = \frac{1}{2}\left(Y_{21} + Y_{12}^{*}\right) = \frac{B}{2}\cdot\frac{1}{s+b} \quad (6)$$

$$H(Y)_{22} = \frac{1}{2}\left(Y_{22} + Y_{22}\right) = g_0 \quad (7)$$

$$R(Y_{11}) = \frac{1}{2}\left(Y_{11} + Y_{11}^{*}\right) = H(Y)_{11} \quad (8)$$

$$R(Y_{12}) = \frac{1}{2}\left(Y_{12} + Y_{12}^{*}\right) = 0 \quad (9)$$

$$R(Y_{21}) = \frac{1}{2}\left(Y_{21} + Y_{21}^{*}\right) = \frac{B}{2}\left(\frac{1}{s+b} + \frac{1}{s^*+b}\right) = B\cdot\frac{Re(s) + b}{|s|^2 + 2b\,Re(s) + b^2} \quad (10)$$

$$R(Y_{22}) = \frac{1}{2}\left(Y_{22} + Y_{22}^{*}\right) = H(Y)_{22}$$

$$H(Y) = \begin{bmatrix} \frac{1}{10}\cdot\frac{|s|^2 + 5.154\times10^{9}\,Re(s) + 3.904\times10^{17}}{|s|^2 + 1.015\times10^{10}\,Re(s) + 2.578\times10^{19}} & 1.925\times10^{10}\,\frac{1}{s^* + 5.077\times10^{9}} \\ 1.925\times10^{10}\,\frac{1}{s + 5.077\times10^{9}} & 2\times10^{-5} \end{bmatrix} \quad (11)$$

$$R(Y) = \begin{bmatrix} \frac{1}{10}\cdot\frac{|s|^2 + 5.154\times10^{9}\,Re(s) + 3.904\times10^{17}}{|s|^2 + 1.015\times10^{10}\,Re(s) + 2.578\times10^{19}} & 0 \\ 3.85\times10^{10}\cdot\frac{Re(s) + 5.077\times10^{9}}{|s|^2 + 1.015\times10^{10}\,Re(s) + 2.578\times10^{19}} & 2\times10^{-5} \end{bmatrix} \quad (12)$$

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Problem 2 a)

$$Z(s) = \frac{2s(s+2)}{(s+8)(s^2+25)} \quad (1)$$

$u = i, \quad y = V \quad (2)$ $\qquad \frac{Y(s)}{U(s)} = Z(s) = \frac{N(s)}{D(s)} \quad (3)$

$$N(s) = 2s^2 + 4 \quad (4)$$

$$D(s) = s^3 + 8s^2 + 25s + 200 \quad (5)$$

$$\begin{cases} U(s) = D(s)\,X(s) & (6) \\ Y(s) = N(s)\,X(s) & (7) \end{cases}$$

define : $x^{(n)} = \frac{d^n x}{dt} \quad (8)$

state variables : $x_1 = x, \quad x_2 = x^{(1)}, \quad x_3 = x^{(2)} \quad (9)$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -200 & -25 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad (10)$$

$$y = \begin{bmatrix} 0 & 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (11)$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -200x_1 - 25x_2 - 8x_3 + u \end{cases} \quad (12)$$

$$y = 4x_2 + 2x_3 \quad (13)$$