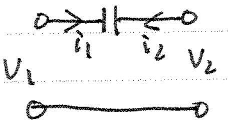
ENEE 610

HW2 J.Y. 09/22/03

Problem 1. a) (15/25)

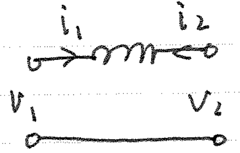
left circuit:



$$i_1 = -i_2 = (V_1 - V_2)C_1 s \qquad (1)$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} C_1 s & -C_1 s \\ -C_1 s & C_1 s \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad (2)$$

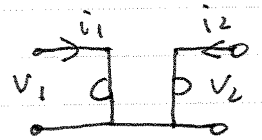
$$Y_C = \begin{bmatrix} C_1 s & -C_1 s \\ -C_1 s & C_1 s \end{bmatrix} \qquad (3)$$



$$i_1 = -i_2 = \frac{V_1 - V_2}{s L_1} \qquad (4)$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{s L_1} & -\frac{1}{s L_1} \\ -\frac{1}{s L_1} & \frac{1}{s L_1} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad (5)$$

$$Y_L = \begin{bmatrix} \frac{1}{s L_1} & -\frac{1}{s L_1} \\ -\frac{1}{s L_1} & \frac{1}{s L_1} \end{bmatrix} \qquad (6)$$

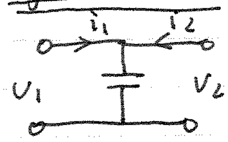


$$Z_g = \begin{bmatrix} 0 & r_1 \\ -r_1 & 0 \end{bmatrix} \qquad (7)$$

$$Y_g = \begin{bmatrix} 0 & -\frac{1}{r_1} \\ \frac{1}{r_1} & 0 \end{bmatrix} \qquad (8)$$

$$\boxed{Y_1 = Y_C + Y_L + Y_g = \begin{bmatrix} C_1 s + \frac{1}{L_1 s} & -C_1 s - \frac{1}{L_1 s} - \frac{1}{r_1} \\ -C_1 s - \frac{1}{L_1 s} + \frac{1}{r_1} & C_1 s + \frac{1}{L_1 s} \end{bmatrix} \qquad (9)}$$

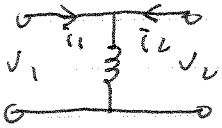
right circuit:



$$V_1 = V_2 = \frac{i_1 + i_2}{Cs} \qquad (10)$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{Cs} & \frac{1}{Cs} \\ \frac{1}{Cs} & \frac{1}{Cs} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \qquad (11)$$

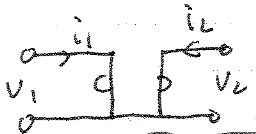
$$Z_C = \begin{bmatrix} \frac{1}{Cs} & \frac{1}{Cs} \\ \frac{1}{Cs} & \frac{1}{Cs} \end{bmatrix} \qquad (12)$$



$$V_1 = V_2 = (i_1 + i_2)Ls \qquad (13)$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Ls & Ls \\ Ls & Ls \end{bmatrix} \qquad (14)$$

$$Z_L = \begin{bmatrix} Ls & Ls \\ Ls & Ls \end{bmatrix} \qquad (15$$



$$Z_g = \begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix} \qquad (16)$$

$$\boxed{Z = Z_C + Z_L + Z_g = \begin{bmatrix} Ls + \frac{1}{Cs} & Ls + \frac{1}{Cs} + r \\ Ls + \frac{1}{Cs} - r & Ls + \frac{1}{Cs} \end{bmatrix} \qquad (17)}$$

For circuit on the left: $\Delta Y_1 = (C_1 s + \frac{1}{L_1 s})^2 - (-C_1 s - L_1 s - \frac{1}{r_1})(-C_1 s - L_1 s - \frac{1}{r_1}) = \frac{1}{r_1^2}$ (1

$$\boxed{Z_1 = Y_1^{-1} = \begin{bmatrix} C_1 r_1^2 s + \frac{r_1^2}{L_1} \cdot \frac{1}{s} & C_1 r_1^2 s + \frac{r_1^2}{L_1} \cdot \frac{1}{s} + r_1 \\ C_1 r_1^2 s + \frac{r_1^2}{L_1} \cdot \frac{1}{s} - r_1 & C_1 r_1^2 s + \frac{r_1^2}{L_1} \cdot \frac{1}{s} \end{bmatrix} \qquad (19)}$$

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HW 2

S.Y. 09/22/03

Problem 1. b) (10/25)

left circuit:

$$Z_1 = \begin{bmatrix} z'_{11} & z'_{12} \\ z'_{21} & z'_{22} \end{bmatrix} = \begin{bmatrix} C_1 r_1^2 s + \frac{r_1^2}{L_1} \frac{1}{s} & C_1 r_1^2 s + \frac{r_1^2}{L_1} \cdot \frac{1}{s} + r_1 \\ C_1 r_1^2 s + \frac{r_1^2}{r_1} \cdot \frac{1}{s} - r_1 & C_1 r_1^2 s \cdot \frac{r_1^2}{L_1} \cdot \frac{1}{s} \end{bmatrix} \quad (20)$$

right circuit:

$$Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} Ls + \frac{1}{C} \cdot \frac{1}{s} & Ls + \frac{1}{C} \cdot \frac{1}{s} + r \\ Ls + \frac{1}{C} \cdot \frac{1}{s} - r & Ls + \frac{1}{C} \cdot \frac{1}{s} \end{bmatrix} \quad (21)$$

$$Z_1 = Z \Rightarrow z'_{11} = z_{11}, \quad z'_{12} = z_{12}, \quad z'_{21} = z_{21}, \quad z'_{22} = z_{22} \quad (22)$$

$$\therefore \begin{cases} C_1 r_1^2 = L & (23) \\ \frac{r_1^2}{L_1} = \frac{1}{C} & (24) \\ r_1 = r & (25) \end{cases}$$

solve r_1, L_1, C_1 in r, L, C

$$\boxed{\begin{cases} r_1 = r & (26) \\ C_1 = \frac{L}{r^2} & (27) \\ L_1 = r^2 C & (28) \end{cases}}$$

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Problem 2. a) (15/25)

$$Z = \begin{bmatrix} Ls+\frac{1}{cs} & Ls+\frac{1}{cs}+r \\ Ls+\frac{1}{cs}-r & Ls+\frac{1}{cs} \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \quad (1)$$

Notice that $z(s) = -z^T(-s)$ lossless (2)

$$z_{in} = \frac{z_{11} z_L + \Delta z}{z_L + z_{22}} \quad (3) \qquad \boxed{z_{in} = \frac{(Ls+\frac{1}{sc})z_L + r^2}{z_L + (Ls+\frac{1}{cs})}}$$

$$Ev\, z_{in}(s) = \frac{1}{2}\left[\frac{z_{11}(s)\, z_L(s) + \Delta z(s)}{z_L(s) + z_{22}(s)} + \frac{z_{11}(-s) z_L(-s) + \Delta z(-s)}{z_L(-s) + z_{22}(-s)}\right] \quad (4)$$

$$N_r(Ev\, z_{in}) = \frac{1}{2}\Big\{[z_{11}(s) z_L(s) + \Delta z(s)][z_L(-s) + z_{22}(-s)]$$

$$[z_{11}(-s) z_L(-s) + \Delta z(-s)][z_L(s) + z_{22}(s)]\Big\}\frac{1}{2}$$

$$= \frac{1}{2}\Big\{\underbrace{z_{11}(s) z_L(-s) z_L(s)}_{①} + z_{11}(s) z_{22}(-s) z_L(s) + \Delta z(s) z_L(-s) + \underbrace{\Delta z(s) z_{22}(-s)}_{③}$$

$$+ \underbrace{z_{11}(-s) z_L(-s) z_L(s)}_{②} + z_{11}(-s) z_{22}(s) z_L(-s) + \Delta z(-s) z_L(s) + \underbrace{\Delta z(-s) z_{22}(s)}_{④}\Big\} \quad (5)$$

z is lossless

$$z_{11}(s) = -z_{11}(-s) \quad (6) \quad z_{22}(s) = -z_{22}(-s) \quad (7)$$

$$\Delta z(-s) = \Delta z^T(-s) = \Delta(-z(s)) = (-1)^n \Delta z(s) \Big|_{n=2} = \Delta z(s) = r^2 \quad (8)$$

terms ① & ② cancel, term ③ & ④ cancel

$$N_r(Ev\, z_{in}) = [-z_{11}(s) z_{22}(s) + \Delta z(s)][z_L(s) + z_L(-s)]/2$$

$$= -z_{12}(s) z_{21}(s) \frac{z_L(s) + z_L(-s)}{2}$$

$$= -z_{12}(s) \cdot z_{21}(s) \cdot Ev\, z_L(s) \quad (9)$$

zeros of Even part of $z_{in}(s)$ are:

$$\boxed{\text{zeros of } z_{12},\ z_{21} \text{ and zeros of even part of } z_L}$$

zeros of z_{12} & z_{21}

$$Ls + \frac{1}{cs} \pm r = 0 \quad (10) \qquad Lcs^2 \pm rcs + 1 = 0 \quad (11)$$

$$\boxed{s = \frac{\pm rc \pm \sqrt{r^2c^2 - 4Lc}}{2Lc}} \quad (12)$$

zeros of z_{21} $\boxed{\frac{rc \pm \sqrt{r^2c^2 - 4Lc}}{2Lc}}$ (13) zeros of z_{12} $\boxed{\frac{-rc \pm \sqrt{r^2c^2 - 4Lc}}{2Lc}}$ (14)

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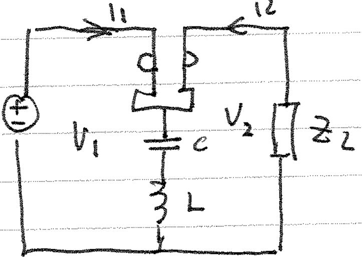
Problem 2. b) (10/25)



$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Ls + \frac{1}{cs} & Ls + \frac{1}{cs} + r \\ Ls + \frac{1}{cs} - r & Ls + \frac{1}{cs} \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \quad (2) \quad \Longrightarrow \quad \begin{aligned} V_1 &= Z_{11} i_1 + Z_{12} i_2 \quad (3) \\ V_2 &= Z_{21} i_1 + Z_{22} i_2 \quad (4) \end{aligned}$$

$$V_2 = - Z_L i_2 \ (5) \quad \Longrightarrow \quad i_2 = - \frac{V_2}{Z_L} \quad (6)$$

$③ \times Z_{21} - ④ \times Z_{11}$

$$Z_{21} V_1 - Z_{11} V_2 = Z_{12} Z_{21} i_2 - Z_{11} Z_{22} i_2 = - \Delta Z \ i_2 \quad (7)$$

Plug (6) in (7)

$$Z_{21} V_1 - Z_{11} V_2 = + \frac{\Delta Z}{Z_2} V_2 \quad (8)$$

$$Z_{21} V_1 = \left(Z_{11} + \frac{\Delta Z}{Z_L} \right) V_2 \quad (9)$$

$$\boxed{\frac{V_2}{V_1} = \frac{Z_{21}}{Z_{11} + \frac{\Delta Z}{Z_2}} = \frac{Z_{21} Z_2}{Z_{11} Z_2 + \Delta Z} \quad (10)}$$

$$Z_{21} = Ls + \frac{1}{cs} - r \quad (11), \qquad Z_{11} = Ls + \frac{1}{cs} \quad (12), \qquad Z_L = \frac{N_r(Z_L)}{D_m(Z_L)} \quad (13), \qquad \delta Z = r^2 \ (1$$

$$\frac{V_2}{V_1} = \frac{\left(Ls + \frac{1}{cs} - r\right) \frac{N_r(Z_L)}{D_m(Z_L)}}{\left(Ls + \frac{1}{cs}\right) \frac{N_r(Z_L)}{D_m(Z_L)} + r^2} = \frac{(Lcs^2 - rcs + 1) N_r(Z_L)}{(Lcs^2 + 1) N_r(Z_L) + r^2 D_m(Z_L)} = 0 \quad (15)$$

zeros of $\frac{V_2}{V_1}$ (zeros of transmission) are zeros of Z_{21} and Load.

$Lcs^2 - rcs + 1 = 0$ $\quad \boxed{\delta = \frac{rc \pm \sqrt{r^2c^2 - 4Lc}}{2Lc} \quad (16)}$

$N_r(Z_L) = 0$ $\quad \boxed{\text{Zeros of load} \quad (17)}$

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HW 2 S.Y. 09/22/08 ⑤

Problem 3. (5 for each synthesis)

$$Z_{in}(s) = \frac{5s(s^2+4)}{(s^2+2)(s^2+8)} \quad (1)$$

a) 1st Foster's

$$Z_{in}(s) = k_\infty s + \frac{k_0}{s} + \frac{2k_1 s}{s^2+2} + \frac{2k_2 s}{s^2+8} \quad (2)$$

$$k_\infty = \frac{1}{s}\cdot Z_{in}(s)\Big|_{s\to\infty} = \frac{5(s^2+4)}{(s^2+2)(s^2+8)}\Big|_{s\to\infty} = 0 \quad (3)$$

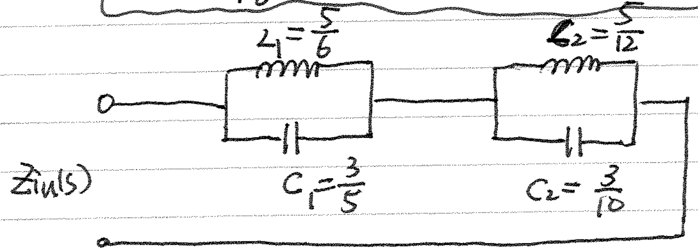
$$k_0 = s\cdot Z_{in}(s)\Big|_{s=0} = \frac{5s^2(s^2+4)}{(s^2+2)(s^2+8)}\Big|_{s=0} = 0 \quad (4)$$

$$2k_1 = \frac{s^2+2}{s}\cdot Z_{in}(s)\Big|_{s^2=-2} = \frac{5(s^2+4)}{s^2+8}\Big|_{s^2=-2} = \frac{5}{3} \quad (5)$$

$$2k_2 = \frac{s^2+8}{s}\cdot Z_{in}(s)\Big|_{s^2=-8} = \frac{5(s^2+4)}{s^2+2}\Big|_{s^2=-8} = \frac{10}{3} \quad (6)$$

$$\therefore Z_{in}(s) = \frac{\frac{5}{3}\cdot s}{s^2+2} + \frac{\frac{10}{3}\cdot s}{s^2+8} \quad (7)$$

$$\boxed{\begin{aligned} C_1 &= \frac{1}{2k_1} = \frac{3}{5} \quad (9), & L_1 &= \frac{2k_1}{\omega_1^2} = \frac{5}{3}\Big/2 = \frac{5}{6} \quad (10) \\ C_2 &= \frac{1}{2k_2} = \frac{3}{10} \quad (11), & L_2 &= \frac{2k_2}{\omega_2^2} = \frac{10}{3}\Big/8 = \frac{5}{12} \quad (12) \end{aligned}}$$



b) 2nd Foster's

$$Y_{in}(s) = \frac{1}{Z_{in}(s)} = \frac{(s^2+2)(s^2+8)}{5s(s^2+4)} \quad (13)$$

$$Y_{in}(s) = k'_\infty s + \frac{k'_0}{s} + \frac{2k'_1 s}{s^2+4} \quad (14)$$

$$k'_\infty = \frac{1}{s}Y_{in}(s)\Big|_{s\to\infty} = \frac{(s^2+2)(s^2+8)}{5s^2(s^2+4)}\Big|_{s\to\infty} = \frac{1}{5} \quad (15)$$

$$k'_0 = s\cdot Y_{in}(s)\Big|_{s=0} = \frac{(s^2+2)(s^2+8)}{5(s^2+4)}\Big|_{s=0} = \frac{4}{5} \quad (16)$$

$$2k'_1 = \frac{s^2+4}{s}\cdot Y_{in}(s)\Big|_{s^2=-4} = \frac{(s^2+2)(s^2+8)}{5s^2}\Big|_{s=-4} = \frac{2}{5} \quad (17)$$

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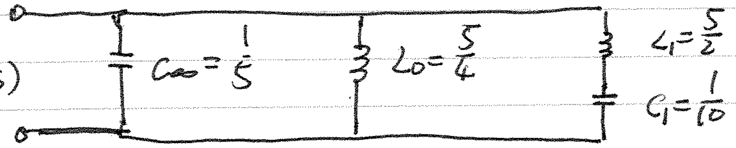
b) (continued)

$$Y_{in}(s) = \frac{1}{5}s + \frac{4}{5}\cdot\frac{1}{s} + \frac{\frac{2}{5}\cdot s}{s^2+4} \quad (18)$$

$$C_\infty = K'_\infty = \frac{1}{5} \quad (19) \qquad L_0 = \frac{1}{K'_0} = \frac{5}{4} \quad (20)$$

$$L_1 = \frac{1}{2K'_1} = \frac{5}{2} \quad (21) \qquad C_1 = \frac{2K'_1}{\omega_1'^2} = \frac{2}{5}\Big/4 = \frac{1}{10} \quad (22)$$



c) 1st Cauer's

$$Z_{in}(s) = \frac{5s(s^2+4)}{(s^2+2)(s^2+8)} = \frac{5s^3+20s}{s^4+10s^2+16} \quad (23) \qquad \text{Order}(Nr(Z_{in})) > \text{Order}(Dn(Z_{in}))$$

$$5s^3+20s \,\big)\, s^4+10s^2+16 \quad \rightarrow \quad \frac{1}{5}s$$

$$-)\ s^4+4s^2$$

$$6s^2+16 \,\big)\, 5s^3+20s \quad \rightarrow \quad \frac{5}{6}s$$

$$-)\ 5s^3+\frac{40}{3}s$$

$$\frac{20}{3}s \,\big)\, 6s^2+16 \quad \rightarrow \quad \frac{9}{10}s \qquad (24)$$

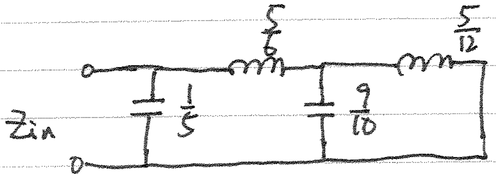
$$-)\ 6s^2$$

$$16 \,\big)\, \frac{20}{3}s \quad \rightarrow \quad \frac{5}{12}s$$

$$-)\ \frac{5}{12}s$$

$$0$$

$$Z_{in}(s) = \cfrac{1}{\frac{1}{5}s + \cfrac{1}{\frac{5}{6}s + \cfrac{1}{\frac{9}{10}s + \cfrac{1}{\frac{5}{12}s}}}} \quad (25)$$



d) 2nd Cauer's

$$Z_{in}(s) = \frac{5s(s^2+4)}{(s^2+2)(s^2+8)} = \frac{20s+5s^3}{16+10s^2+s^4} \qquad (26)$$

$$\begin{array}{l}
\qquad\qquad\qquad \frac{4}{5}s^{-1} \\
20s+5s^3 \, \big) \, \overline{16+10s^2+s^4} \\
\quad -)\ \underline{16+4s^2} \qquad\qquad \frac{10}{3}s^{-1} \\
\qquad\qquad 6s^2+s^4 \, \big) \, \overline{20s+5s^3} \\
\qquad\qquad\quad -)\ \underline{20s+\frac{10}{3}s^3} \qquad \frac{18}{5}s^{-1} \\
\qquad\qquad\qquad\qquad \frac{5}{3}s^3 \, \big) \, \overline{6s^2+s^4} \\
\qquad\qquad\qquad\qquad\quad -)\ \underline{6s^2} \qquad \frac{5}{3}s^{-1} \\
\qquad\qquad\qquad\qquad\qquad s^4 \, \big) \, \overline{\frac{5}{3}s^3} \\
\qquad\qquad\qquad\qquad\qquad\quad -)\ \underline{\frac{5}{3}s^3} \\
\qquad\qquad\qquad\qquad\qquad\qquad 0
\end{array} \qquad (27)$$

$$Y_{in}(s) = \frac{1}{Z_{in}(s)} = \frac{16+10s^2+s^4}{20s+5s^3} \qquad (28)$$

$$\boxed{\begin{aligned} Y_{in}(s) &= \frac{4}{5}s^{-1} + \cfrac{1}{\frac{10}{3}s^{-1} + \cfrac{1}{\frac{18}{5}s^{-1} + \cfrac{1}{\frac{5}{3}s^{-1}}}} \\ &= \frac{1}{\frac{5}{4}s} + \cfrac{1}{\frac{1}{\frac{3}{10}s} + \cfrac{1}{\frac{1}{\frac{5}{18}s} + \cfrac{1}{\frac{1}{\frac{3}{5}s}}}} \end{aligned}} \qquad (29)$$