file: c:\jiangyu\Courses_T\enee610\Fall03\HomeWork\HW1_2a.mcd JY 09/16/03
HW1: Problem2 solution (synthesis using Richards function)

$A := 4 \quad a := 1 \quad b := 2 \quad c := 3 \quad d := 5$

$$z(s) := \frac{A\cdot((s+a)\cdot(s+c))}{((s+b)\cdot(s+d))} \qquad Nz(s) := s^2 + (a+c)\cdot s + a\cdot c \qquad Dz(s) := s^2 + (b+d)\cdot s + b\cdot d$$

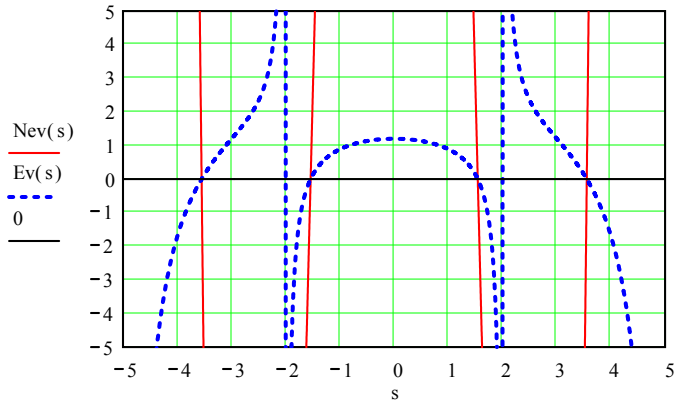
$$Ev(s) := \frac{(z(s)+z(-s))}{2} \qquad Nev(s) := Nz(s)\cdot Dz(-s) + Nz(-s)\cdot Dz(s)$$

To plot to see where zeros of even part may be

$smax := 5 \quad sinc := 0.001 \quad smin := -smax$

$Emax := 5 \quad Emin := -Emax$

$s := smin, smin + sinc .. smax$



To solve more precisely for zeros of the even part use trials at s=0.6 and s=2.0

$s := 1.53$

$r1 := root(Nev(s), s)$ $\quad r1 = 1.54159807 \quad r1^2 = 2.37652461$

$s := 3.6$

$r2 := root(Nev(s), s)$

$r2 = 3.55295305 \quad r2^2 = 12.62347538$

$k := r1 \quad z(k) = 1.99293 \quad Nev(r1) = 7.07322485\cdot10^{-8} \quad Ev(r1) = 3.85161159\cdot10^{-9}$

$Nev(-r1) = 7.07322485\cdot10^{-8} \quad Nev(r2) = 7.87366616\cdot10^{-10}$

More precise evaluation in this case:

$B := b\cdot d + a\cdot c - (a+c)\cdot(b+d) \quad B = -15$

$C := a\cdot b\cdot c\cdot d \quad C = 30$

$$r1 := \sqrt{\frac{-B}{2} - \left(\frac{1}{2}\right)\cdot\sqrt{(B^2 - 4\cdot C}} \qquad r1 = 1.54159807$$

$$r2 := \sqrt{\left(\frac{-B}{2}\right) + \left(\frac{1}{2}\right)\cdot\sqrt{(B^2 - 4\cdot C}} \qquad r2 = 3.55295305$$

$k := r1 \qquad k1 := r1$

For the richards function $R(s,k) := \dfrac{(k \cdot z(s) - s \cdot z(k))}{(k \cdot z(k) - s \cdot z(s))}$ we are interested in

$$Nr(s) := (k \cdot A \cdot (s+a) \cdot (s+c) - s \cdot z(k) \cdot (s+b) \cdot (s+d))$$

$$Nr(s) := -z(k) \cdot s^3 + (k \cdot A - (b+d) \cdot z(k)) \cdot s^2 + (k \cdot A \cdot (a+c) - z(k) \cdot b \cdot d) \cdot s + k \cdot A \cdot a \cdot c$$

$n3 := -z(k) \qquad n3 = -1.99293177 \qquad n0 := k \cdot A \cdot a \cdot c$

$n2 := k \cdot A - (b+d) \cdot z(k) \qquad n2 = -7.78413013 \qquad n0 = 18.49917687$

$n1 := k \cdot A \cdot (a+c) - z(k) \cdot b \cdot d \qquad n1 = 4.73625142$

$$Nr(s) := -1.99293177 \cdot s^3 + -7.78413013 \cdot s^2 + 4.73625142 \cdot s + 18.49917687$$

$$\frac{n2}{n3} = 3.90586885 \qquad \frac{n1}{n3} = -2.37652462 \qquad \frac{n0}{n3} = -9.28239346$$

$$Nr(s) := -1.99293177 \cdot \left(s^3 + 3.90586885 \cdot s^2 + -2.37652462 \cdot s + -9.28239346\right)$$

$$NRi(s) := \frac{Nr(s)}{s^2 - 2.37652461} \qquad zeronum := -3.90586885$$

$zeronum = -3.90586885$

$$Dr(s) := k \cdot z(k) \cdot (s+b) \cdot (s+d) - s \cdot A \cdot (s+a) \cdot (s+c)$$

$$Dr(s) := -A \cdot s^3 + (k \cdot z(k) - A \cdot (a+c)) \cdot s^2 + (k \cdot z(k) \cdot (b+d) - A \cdot a \cdot c) \cdot s + k \cdot z(k) \cdot b \cdot d$$

$d3 := -A \qquad d3 = -4 \qquad d0 := k \cdot z(k) \cdot b \cdot d$

$d2 := k \cdot z(k) - A \cdot (a+c) \qquad d2 = -12.92770022 \qquad d0 = 30.72299781$

$d1 := k \cdot z(k) \cdot (b+d) - A \cdot a \cdot c \qquad d1 = 9.50609847$

$$Dr(s) := -4 \cdot s^3 + -12.92770022 \cdot s^2 + 9.50609847 \cdot s + 30.72299781$$

$$\frac{d2}{d3} = 3.23192505 \qquad \frac{d1}{d3} = -2.37652462 \qquad \frac{d0}{d3} = -7.68074945$$

$$Dr(s) := -4 \cdot \left(s^3 + 3.23192505 \cdot s^2 - 2.37652462 \cdot s - 7.68074945\right)$$

$zerodem := -3.23192505$

$zerodem = -3.23192505$

$Nr(r1) = -1.65089072 \cdot 10^{-8} \qquad Nr(-r1) = -3.481858 \cdot 10^{-8} \qquad Nr(r2) = -152.32013549$

$Dr(r1) = 5.16176328 \cdot 10^{-8} \qquad Dr(-r1) = 1.48684158 \cdot 10^{-8} \qquad Dr(r2) = -278.09724734$

$$\frac{n3}{d3} = 0.49823294$$

Therefore R(s,k):= $\dfrac{k \cdot z(s) - s \cdot z(k)}{k \cdot z(k) - s \cdot z(s)} = \left\langle \dfrac{n3}{d3} \right\rangle$ (s-zerinum)/(s-zerodem))

$$R(s,k) := 0.49823294 \cdot \frac{\left[\left(s^2 - 2.37652461\right)\cdot(s + 3.90586885)\right]}{\left[\left(s^2 - 2.37652461\right)\cdot(s + 3.23192505)\right]}$$

$rgyr1 := z(k)$ $\quad rgyr1 = 1.99293177$ $\quad C1 := \frac{1}{k \cdot z(k)}$ $\quad C1 = 0.32548907$

$$KL := \left(\frac{n3}{d3}\right)\cdot z(k) \qquad KL = 0.99294426$$

$e := -zeronum$ $\quad f := -zerodem$

$$zL(s) := KL \cdot \frac{(s + e)}{(s + f)}$$

zL(s) is the load on the first lossless coupling section. To repeat on it.

$$EvzL(s) := \frac{(zL(s) + zL(-s))}{2} \qquad EvzL(r2) = 2.11983775 \cdot 10^{-9}$$

$$NzLEv(s) := KL \cdot \frac{((s + e)\,(-s + f) + (-s + e)\,(s + f))}{2}$$

$$NzLEv(s) := KL \cdot \left(-s^2 + e \cdot f\right) \qquad e \cdot f = 12.62347538$$

$zL(r3) = 1.09157369$ $\quad zL(-r3) = -1.09157369$ $\quad r3 := \sqrt{e \cdot f}$ $\quad r3 = 3.55295305$

$EvzL(r3) = 0$

Forming Richards' function of zL(s): $RL(s,k) := \frac{(k \cdot zL(s) - s \cdot zL(k))}{(k \cdot zL(k) - s \cdot zL(s))}$

In the following KL cancels numerator and denominator so is dropped

$k2 := r3$

$$NRL(s) := k2 \cdot (s + e) \cdot (k2 + f) - s \cdot (k2 + e) \cdot (s + f)$$

$$NRL(s) := -(k2 + e) \cdot s^2 + -((k2 \cdot (k2 + f) - (k2 + e) \cdot f) \cdot s + k2 \cdot e \cdot (k2 + f)$$

$nR2 := -(k2 + e)$ $\quad nR2 = -7.4588219$

$nR1 := (k2 \cdot (k2 + f) - (k2 + e) \cdot f)$ $\quad nR1 = 0$

$nR0 := k2 \cdot e \cdot (k2 + f)$ $\quad nR0 = 94.15625461$

$$\frac{nR0}{nR2} = -12.62347538$$

$$DRL(s) := k2 \cdot (k2 + e) \cdot (s + f) - s \cdot (k2 + f) \cdot (s + e)$$

$$DRL(s) := -(k2 + f) \cdot s^2 + (k2 \cdot (k2 + e) - e \cdot (k2 + f)) \cdot s + (k2 \cdot (k2 + e) \cdot f$$

$dR2 := -(k2 + f)$ $\quad dR2 = -6.7848781$

$dR1 := k2\cdot(k2+e) - e\cdot(k2+f)$ $\quad dR1 = 3.55271368\cdot10^{-15}$

$dR0 := (k2\cdot(k2+e)\cdot f$ $\quad dR0 = 85.64874165$

$$\frac{dR0}{dR2} = -12.62347538$$

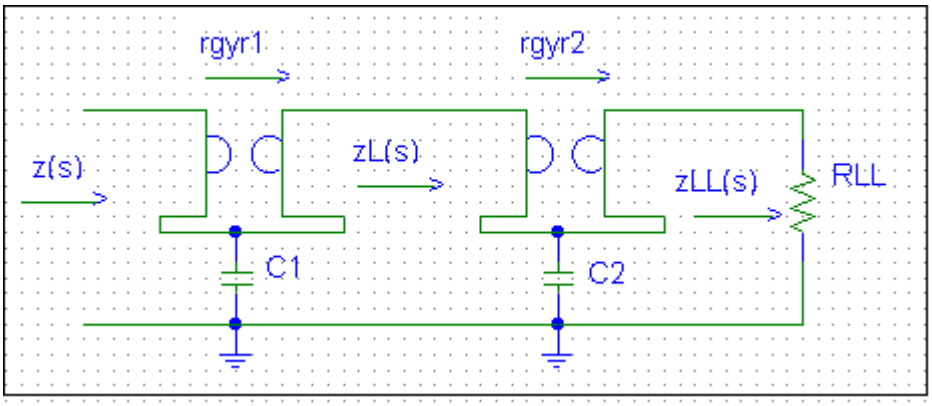
$$zLL(s) := zL(k2)\cdot\frac{\left[KL\cdot nR2\cdot\left(s^2 - \frac{nR0}{nR2}\right)\right]}{\left[KL\cdot dR2\cdot\left(s^2 - \frac{dR0}{dR2}\right)\right]}$$

$$\frac{nR2}{dR2} = 1.09933027 \qquad zL(k2) = 1.09157369$$

$zLL(s) := zL(k2)\cdot\frac{nR2}{dR2}$ $\quad zLL(s) = 1.2$ $\quad RLL := zLL(s)$

$rgyr2 := zL(k2)$ $\quad rgyr2 = 1.09157369$

$C2 := \frac{1}{k2\cdot zL(k2)}$ $\quad C2 = 0.25784426$



$k1 = 1.54159807$ $\quad k2 = 3.55295305$

$z(s) := \frac{4\cdot((s+1)\,(s+2))}{(s+2)\cdot(s+5)}$ $\quad zL(s) := 0.99294426\cdot\frac{(s+3.90586885)}{(s+3.23192505)}$ $\quad RLL = 1.2$

$rgyr1 = 1.99293177$ $\quad rgyr2 = 1.09157369$

$C1 = 0.32548907$ $\quad C2 = 0.25784426$