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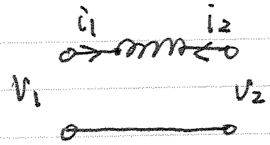
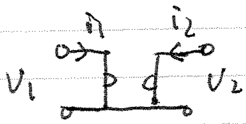
HW1 J.Y. 09/15/03

Problem 1. a) (10/30)



$$i_1 = -i_2 = (V_1 - V_2)/sL \quad (1) \qquad Y_L = \begin{bmatrix} \frac{1}{sL} & -\frac{1}{sL} \\ -\frac{1}{sL} & \frac{1}{sL} \end{bmatrix} \quad (2)$$



$$Z_g = \begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix} \quad (3) \qquad Y_g = Z_g^{-1} = \begin{bmatrix} 0 & -\frac{1}{r} \\ \frac{1}{r} & 0 \end{bmatrix} \quad (4)$$

$$Y = Y_g + Y_L = \begin{bmatrix} \frac{1}{sL} & -\frac{1}{sL} - \frac{1}{r} \\ -\frac{1}{sL} + \frac{1}{r} & \frac{1}{sL} \end{bmatrix} \quad (5) \qquad \Delta Y = \left(\frac{1}{sL}\right)^2 - \left(-\frac{1}{sL} - \frac{1}{r}\right)\left(-\frac{1}{sL} + \frac{1}{r}\right) = \frac{1}{r^2} \quad (6)$$

$$Z = Y^{-1} = r^2 \begin{bmatrix} \frac{1}{sL} & \frac{1}{sL} + \frac{1}{r} \\ \frac{1}{sL} - \frac{1}{r} & \frac{1}{sL} \end{bmatrix} = \begin{bmatrix} \frac{r^2}{sL} & \frac{r^2}{sL} + r \\ \frac{r^2}{sL} - r & \frac{r^2}{sL} \end{bmatrix} \quad (7)$$

$$\Delta Z = \left(\frac{r^2}{sL}\right)^2 - \left(\frac{r^2}{sL} + r\right)\left(\frac{r^2}{sL} - r\right) = r^2 = 1/\Delta Y \quad (8)$$

$$Z_{in} = \frac{Z_{11} Z_L + \Delta Z}{Z_L + Z_{22}} = \frac{\frac{r^2}{sL} Z_L + r^2}{Z_L + \frac{r^2}{sL}} = \frac{r^2 Z_L + r^2 sL}{sL Z_L + r^2} \quad (9)$$

b) (15/30)

$$Z_L = \frac{\Delta Z - Z Z_{22}}{Z - Z_{11}} = \frac{r^2 - Z\frac{r^2}{sL}}{Z - \frac{r^2}{sL}} = \frac{sL - Z}{\left(\frac{sL}{r^2}\right) Z - 1} \quad (10)$$

$$\text{Rich} = \frac{kZ(s) - sZ(k)}{kZ(k) - sZ(s)} = \frac{\frac{s}{k} - \frac{Z(s)}{Z(k)}}{\frac{s}{k}\cdot\frac{Z(s)}{Z(k)} - 1} = \frac{1}{Z(k)} \cdot \frac{\frac{Z(k)}{k}\cdot s - Z}{\frac{1}{Z(k)\cdot k}\cdot s \cdot Z - 1} \quad (11)$$

$$Z_L = \frac{L \cdot s - Z}{\left(\frac{L}{r^2}\right) s \cdot Z - 1} \quad (12)$$

Compare (10) with (12) $\Rightarrow \begin{cases} \frac{Z(k)}{k} = L & (13) \\ \frac{1}{kZ(k)} = \frac{L}{r^2} & (14) \end{cases} \Rightarrow \begin{cases} r = Z(k) & (15) \\ L = \frac{Z(k)}{k} & (16) \end{cases}$

$$\therefore \boxed{Z_L = r \cdot \left[\frac{1}{r} \cdot \frac{L \cdot s - Z}{\left(\frac{L}{r^2}\right)\cdot s \cdot Z - 1}\right] = r \cdot \text{Rich} \quad (17)}$$

c) (5/30)

$$\boxed{\begin{aligned} r &= Z(k) & (18) \\ L &= \frac{Z(k)}{k} & (19) \end{aligned}}$$

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Problem 2

a) (10/30) b) (15/30)

c) (5/30)

Four zeros of even part of z_{in} are $S_{1,2} = \pm k_1$, $S_{3,4} = \pm k_2$, $(k_1, k_2 > 0)$ (1)

when $k = k_1 > 0$

$$\begin{cases} r = z(k_1) > 0 & (2) \\ c = \dfrac{1}{k_1 \delta(k_1)} > 0 & (3) \\ Z_L = r \cdot Rich(k_1, S) = z(k_1)\, Rich(k_1, S) & (4) \\ Rich(k_1, s) = \dfrac{k_1 z(S) - S z(k_1)}{k_1 z(k_1) - S z(S)} & (5) \end{cases}$$

if $k = -k_1 < 0$

since $z(k_1) + z(-k_1) = 0$ (6) $\Longrightarrow$ $z(-k_1) = -z(k_1)$ (7)

$$r' = z(k)\Big|_{k=-k_1} = z(-k_1) = -z(k_1) = -r \quad (8)$$

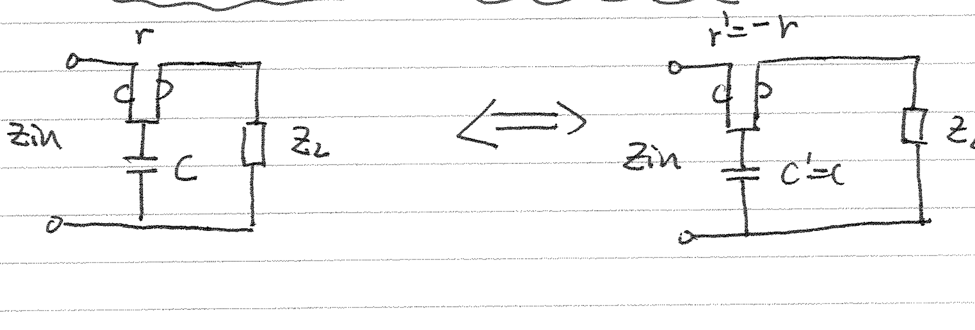
$$c' = \frac{1}{k\delta(k)}\Big|_{k=-k_1} = \frac{1}{(-k_1)\delta(-k_1)} = \frac{1}{k_1 \delta(k)} = C \quad (9)$$

$$Rich(-k_1, s) = Rich(k, s)\Big|_{k=-k_1} = \frac{(-k_1) z(S) - S z(-k_1)}{(-k_1) z(-k_1) - S z(S)} = -\frac{k_1 z(S) - S z(k_1)}{k_1 z(k_1) - S z(S)} = -Rich(k_1, S) \quad (10)$$

$$Z_L' = z(k)\, Rich(k, S)\Big|_{k=-k_1} = z(-k_1)\, Rich(-k_1, S) = z(k_1) \cdot Rich(k_1, S) = Z_L \quad (11)$$

for $k = -k_1 < 0$

$$\boxed{\therefore \begin{cases} r' = -r < 0 & (12) \\ c' = C > 0 & (13) \\ Z_L' = Z_L & (14) \end{cases}}$$ inverse gyrator.



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Problem 3 (extra credit)

(5/10)

a) $Zin(s) = \frac{4(s+1)(s+3)}{(s+2)(s+5)} = \frac{A(s+a)(s+c)}{(s+b)(s+d)}$ (1) $(0<a<b<c<d)$

$Ev(Zin) = \frac{1}{2}(Zin(s) + Zin(-s))$ (2)

$Nv(s) = s^4 - [(a+c)(b+d) - bd - ac]s^2 + abcd = 0$ (3)

$B = (a+c)(b+d) - bd - ac$ (4)

$C = abcd$ (5)

$Nv(s) = s^4 - Bs^2 + C = 0$ (6)

$B = ab + a(d-c) + cb + (c-b)d > 0$ (7)

$C = abcd > 0$ (8)

$\Delta = B^2 - 4C > 0$ with $0<a<b<c<d$ (9)

$t = s^2$, the $t^2 - Bt + C = 0$ (10)

$\Delta = B^2 - 4C > 0$, two real solutions t_1 & t_2 of equation (10)

$$\begin{cases} B = t_1 + t_2 > 0 & (11) \\ C = t_1 \cdot t_2 > 0 & (12) \\ t_1, t_2 \text{ are real} & (13) \end{cases} \Rightarrow t_1 > 0,\ t_2 > 0 \quad (14)$$

the $s_{1,2,3,4}^2 = t_{1,2}$ $\Rightarrow$ $\boxed{s_{1,2,3,4} \text{ are real} \quad s_{1,2} = \pm\sqrt{t_1},\ s_{3,4} = \pm\sqrt{t_2}}$ (15)

$\therefore$ there are two real positive zeros of even part of $Zin(s)$

$Order(Dm(Zin)) = Order(Nm(Zin)) = 2$

$\therefore$ enough real positive zeros.

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Problem 3 b) (5/10)

$$Z_L(S) = Z(k) \cdot \text{Rich} = Z(k)\,\frac{kZ(S) - SZ(k)}{kZ(k) - SZ(S)}, \qquad Z_{in} = Z(S) \quad (1)$$

$$\text{Even}[Z_L(S)] = \frac{1}{2}[Z_L(S) + Z_L(-S)]$$

$$= \frac{1}{2}\left\{\frac{kZ(S) - SZ(k)}{kZ(k) - SZ(S)} + \frac{kZ(-S) + SZ(k)}{kZ(k) + SZ(-S)}\right\} = \frac{1}{2}\,\frac{N_r(E_v Z_L)}{D_m(E_v Z_L)} \quad (2)$$

$$D_m(E_v Z_L) = [kZ(k) - SZ(S)][kZ(k) + SZ(-S)] \quad (3)$$

$$N_r(E_v Z_L) = k^2 Z(k) Z(S) + kS\,Z(S)Z(-S) - kZ^2(k)S - Z(k)S^2 Z(-S)$$

$$+ k^2 Z(k) Z(-S) - kS\,Z(S)Z(-S) + kZ^2(k)S - Z(k)S^2 Z(S)$$

$$= (k^2 - S^2)[Z(S) + Z(-S)] \quad (4)$$

$\therefore$ zeros of Even$[Z_L(S)]$ are the zeros of $N_r(E_v Z_L)$ and poles of $D_m(Z_v Z_L)$

i) zeros of $N_r(E_v Z_L)$:

$S = \pm k$ seems to be the zeros

But, since $D_m(E_v Z_L) = [kZ(k) - SZ(S)][kZ(k) - SZ(-S)]\Big|_{S=\pm k} = 0$

$D_m(E_v Z_L)$ has at least order 1 of zeros at $S = \pm k$ which cancel zeros at $S = \pm k$ of $N_r(E_v Z_L)$

$S = \pm k$ are not the zeros of Even$[Z_L(S)]$

$\therefore$ zeros of $N_r(E_v Z_L)$ are the zeros of even part of $Z(S)$ $(Z_{in}(S))$

ii) poles of $D_m(Z_v Z_L)$

$$Z_L = \frac{N_r(Z\ )}{D_m(Z\ )} \quad (5)$$

$D_m(E_v Z_L)$ has poles at $D_m(Z(S)) \cdot D_m(Z(-S)) = 0$

But $[Z(S) + Z(-S)]$ also has poles at $D_m(Z(S)) \cdot (D_m Z(-S)) = 0$ which cancel

$\therefore$ poles of $D_m(Z_v Z_L)$ are not the zeros of Even$[Z_L(S)]$

$\therefore$ Zeros of Even part of $Z_L(S)$ can only be zeros of Even Part of $Z(S)$ $(= Z_{in}(S))$.