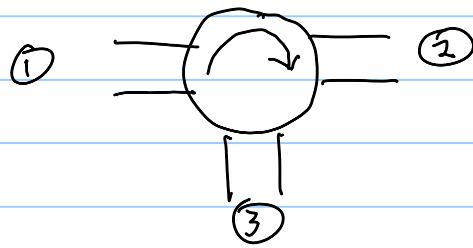
EE610_11/03/03

Note Title 11/3/2003

circulator — 3 port



$$S = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} ; \quad V^{r} = S V^{i}$$

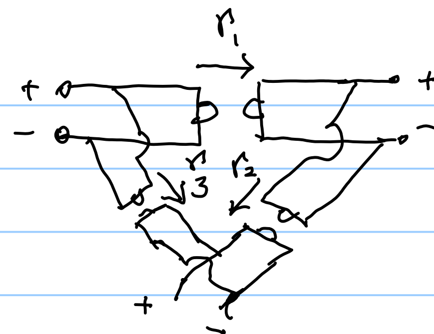
$$S^T S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \frac{1}{3}$$

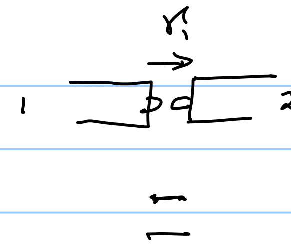
lossless

p. 2

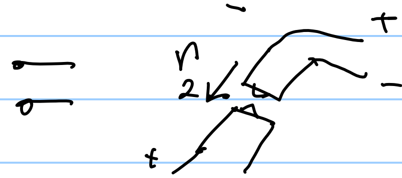
EE610

11/03/03

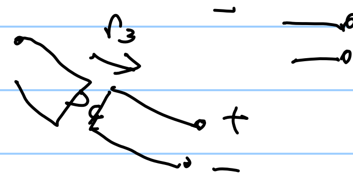




$$Y_1 = \begin{bmatrix} 0 & -g_1 & 0 \\ g_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



$$Y_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -g_2 \\ 0 & g_2 & 0 \end{bmatrix}$$



$$Y_3 = \begin{bmatrix} 0 & 0 & -g_3 \\ 0 & 0 & 0 \\ g_3 & 0 & 0 \end{bmatrix}$$

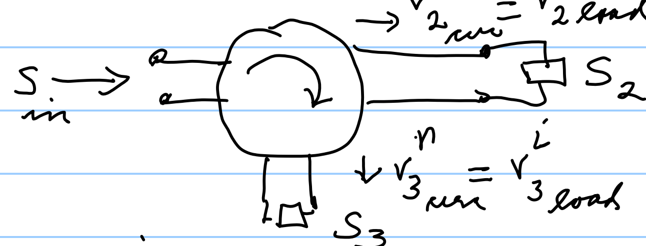
$$S = (1_3 + Y)^{-1}(1_3 - Y)\;;\quad Y = Y_1 + Y_2 + Y_3 = \begin{bmatrix} 0 & -g_1 & -g_3 \\ g_1 & 0 & -g_2 \\ g_3 & g_2 & 0 \end{bmatrix}$$

$$S = \frac{1}{1+g_1^2+g_2^2+g_3^2}\begin{bmatrix} 1+g_2^2-g_1^2-g_3^2 & 2g_1\left(1-\frac{g_2g_3}{g_1}\right) & 2g_3\left(1+\frac{g_1g_2}{g_3}\right) \\ -2g_1\left(1+\frac{g_2g_3}{g_1}\right) & 1-g_1^2-g_2^2+g_3^2 & 2g_2\left(1-\frac{g_1g_3}{g_2}\right) \\ -2g_3\left(1-\frac{g_1g_2}{g_3}\right) & -2g_2\left(1+\frac{g_1g_3}{g_2}\right) & 1+g_1^2-g_2^2-g_3^2 \end{bmatrix}$$

set $g_1 = g_2 = -1$, $g_3 = 1$

$$S = \frac{1}{4}\begin{bmatrix} 0 & 0 & 4 \\ 4 & 0 & 0 \\ 0 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Put loads on



$\rightarrow V^r_{2\,circ} = V^i_{2\,load}$

$\downarrow V^r_{3\,circ} = V^i_{3\,load}$

$S_{in} \Rightarrow V^r_1 = S_{in} V^i_1$

$V^r_1 = V^i_3 \; ; \; V^r_3 = V^i_2$ for circulator

$V^i_{3\,load} = V^r_{3\,circulator}$

$V^r_{3\,load} = V^i_3 = S_3 V^i_{3\,load} = S_3 V^r_3$

$V^r_{1\,circ} = V^i_{3\,circ} = S_3 V^r_3 =$

$= S_3 V^i_{2\,circ} = S_3 V^r_{2\,load}$

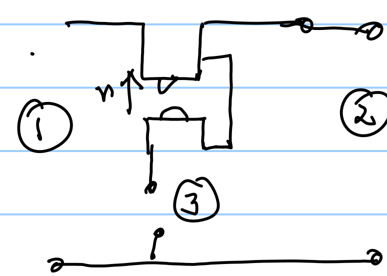
$V^r_{2\,load} = S_2 V^i_{2\,load} = S_2 V^r_{2\,circ}$

$= S_2 V^i_{1\,circ}$ (def of circulator)

$= S_3 \cdot S_2 \cdot V^i_{1\,circ} \Longrightarrow S_{in} = S_3 \cdot S_2$

as circulator is passive, if the two loads are passive so is the circuit $\Rightarrow$ product of bounded real functions, S_2, S_3, is S_{in} bounded real (not true for positive real functions).

can do with one gyrator:



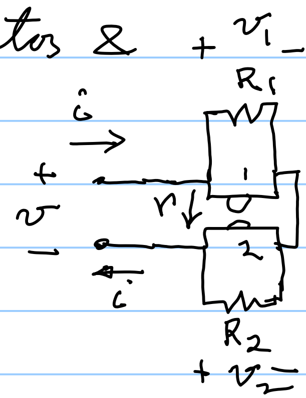
$$S = \frac{1}{3+r^2}\begin{bmatrix} r^2-1 & 2-2r & 2+2r \\ 2+2r & r^2-1 & 2-2r \\ 2-2r & 2+2r & r^2-1 \end{bmatrix}$$

if $r=1$ $\quad S = \frac{1}{4}\begin{bmatrix} 0 & 0 & 4 \\ 4 & 0 & 0 \\ 0 & 4 & 0 \end{bmatrix}$

EE610
11/03/03
p.6

norators

nullators &



$$v = v_1 - v_2$$

$$i = G_1 v_1 + i_{g_1} = -G_2 v_2 - i_{g_2}$$

$$v_1 = r i_{g_2} \; ; \; v_2 = -r i_{g_1}$$

$$\Rightarrow v = r(i_{g_2} + i_{g_1})$$

$$i = G_1 r i_{g_2} + i_{g_1} = +G_2 r i_{g_1} - i_{g_2}$$

$$(G_1 r + 1) i_{g_2} = (G_2 r - 1) i_{g_1}$$

if $i_{g_2} = -i_{g_1} \Rightarrow v = 0$ & $0 = [(G_1 r + 1) + (G_2 r - 1)] i_{g_1}$
$= r(G_1 + G_2) i_{g_1}$

if also $G_1 \neq -G_2 \Rightarrow i_{g_1} = 0 = -i_{g_2} = 0 \Rightarrow v_1 = v_2 = 0$

$\Rightarrow i = 0, \; v = 0$ can also guarantee by $G_1 r + 1 = 0$

EE610
11/03/03
p. 7

then $G_1 = -\frac{1}{r}$; $G_2 = -G_1 \Rightarrow$ nullator

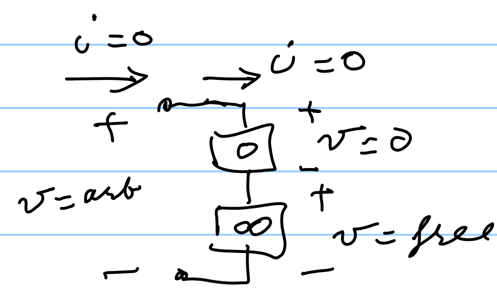
if $i_{g_2} = i_{g_1} \Rightarrow v = 2r\, i_{g_1}$, $i = (1+Gr)\, i_{g_1} \Rightarrow$ allows i_{g_1} to get anywhere

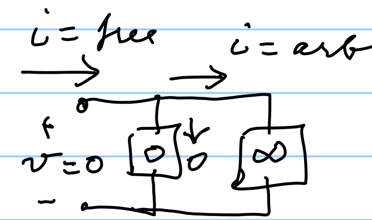
$G_1 r + 1 = 0$, $G_2 r - 1 = 0 \Rightarrow i_{g_1}$ & i_{g_2} can be anything

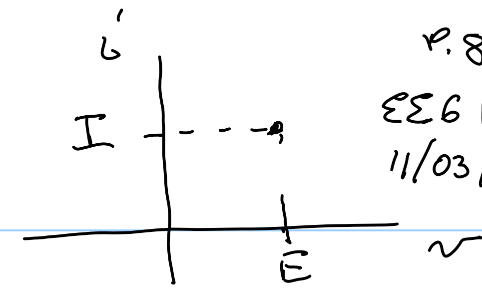
$\Rightarrow i$ = arbitrary

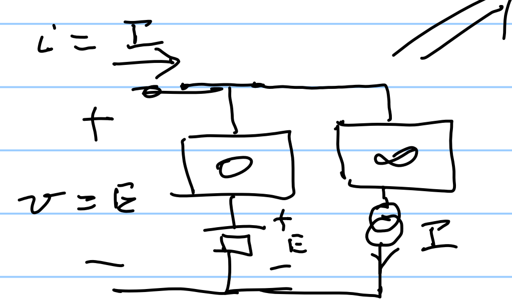
v = arbitrary independent of i

$\Rightarrow$ norator









allows only $v = E$
$i = I$

Bilateral Laplace transform $f(t)$

$$\mathcal{L}[f] = \int_{-\infty}^{\infty} f(t)\, e^{-\alpha t}\, dt = \langle f(t), e^{-\alpha t} \rangle_t$$

if $f(t) = \delta(t) = \langle \delta, e^{-\alpha t} \rangle_t \doteq e^{-\alpha 0} = 1$

for $e^{at} 1(-t) \Rightarrow \int_{-\infty}^{\infty} e^{at} e^{-st} 1(-t)\, dt$

$$= \int_{-\infty}^{0} e^{(a-s)t}\, dt = \frac{1}{a-s} e^{(a-s)t}\Big|_{-\infty}^{0}$$

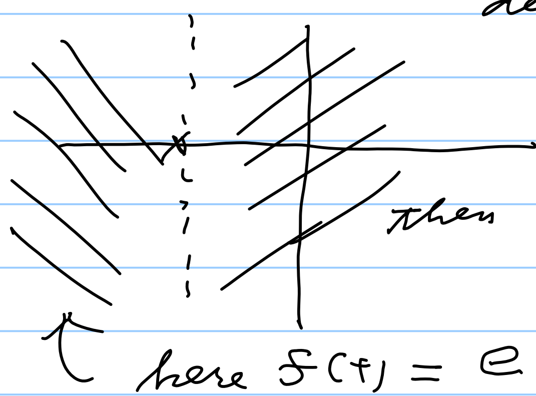
$$= \frac{1}{a-s} - \underbrace{\frac{1}{a-s} e^{(a-s)t}}_{\text{needs } Re(a-s) > 0} \quad t \to -\infty$$

$\therefore \; \mathcal{L}[e^{at} 1(-t)] = \frac{-1}{s-a}$ if $\underbrace{Re\, s < Re\, a}_{\text{region of convergence}}$

a left half plane

$\mathcal{L}[-e^{at} 1(t)] = \frac{-1}{s-a}$ if $Re\, s > Re\, a \Rightarrow$ a RHP

$F(s) = \frac{-1}{s-a}$ need region of convergence to determine $f(t)$



then $f(t) = -e^{at} 1(t)$

here $f(t) = e^{at} 1(-t)$