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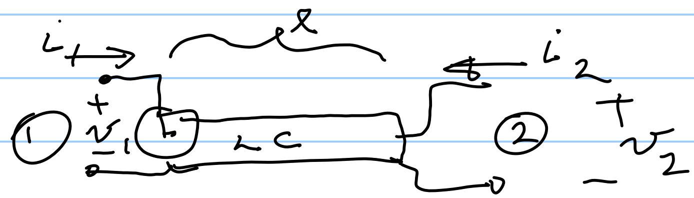
Note Title 10/29/2003



$\Rightarrow$

$$z_0 = \sqrt{L/C}$$

$$Z(s) = z_0 \begin{bmatrix} \operatorname{ctnh}(\sqrt{LC}\,\ell s) & \operatorname{csch}(\sqrt{LC}\,\ell s) \\ \operatorname{csch}(\sqrt{LC}\,\ell s) & \operatorname{ctnh}(\sqrt{LC}\,\ell s) \end{bmatrix}$$

$$\operatorname{ctnh}(x) = \frac{1}{x} + 2x \sum_{i=1}^{\infty} \frac{1}{x^2 + (i\pi)^2}$$

poles @ $x = \pm j i\pi,\ 0$

$i = 1, 2, \dots,$

$$\operatorname{csch}(x) = \frac{1}{x} + 2x \sum_{i=1}^{\infty} \frac{(-1)^i}{x^2 + (i\pi)^2}$$

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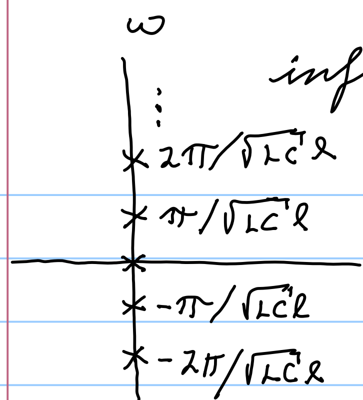
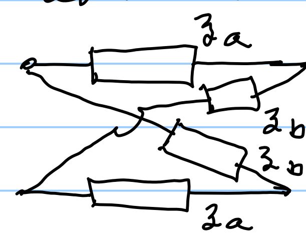


infinite number of poles
by choosing different lengths l can put at any point. If choose $\pi/(\sqrt{LC}\,l)$ rational and lines in series (as 2-ports) can get a pole at every rational ω with irrationals as essential singularities (as limits of poles).

— constant R:

Lattice



$$Z = \frac{1}{2}\begin{bmatrix} z_b + z_a & z_b - z_a \\ z_b - z_a & z_b + z_a \end{bmatrix}$$

$$V_2 = -RI_2 = z_{21}I_1 + z_{22}I_2 \Rightarrow I_2 = \frac{-z_{21}}{R + z_{22}} I_1$$

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$$V_1 = z_{11} I_1 + z_{12} I_2 = \left(z_{11} - \frac{z_{12} z_{21}}{R + z_{22}}\right) I_1$$

or $$\frac{V_1}{I_1} = \frac{\Delta_z + R z_{11}}{R + z_{22}} = z_{in} = R \text{ if constant } R$$

$$\Rightarrow \Delta_z + \cancel{R z_{11}} = R^2 + \cancel{R z_{22}} \quad \text{as } z_{11} = z_{22}$$

for lattice

$$\Delta_z = \frac{1}{4}\left((z_b + z_a)^2 - (z_b - z_a)^2\right) = z_a z_b$$

$$\frac{V_2}{V_1} = -\frac{R I_2}{R I_1}\Bigg|_{\text{if constant } R} = \frac{z_{21}}{R + z_{22}}$$

$$\Rightarrow z_a z_b = R^2 \Rightarrow z_b = R^2 / z_a$$

(note if z_a is positive real so is $z_b = R^2/z_a$)

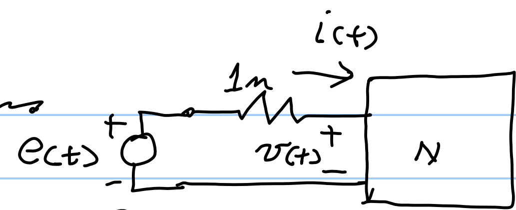
$\therefore$ choose $z_b = R^2/z_a$ & get $z_{in} = R$

$$\frac{V_2}{V_1} = \frac{\frac{1}{2}(z_b - z_a)}{R + \frac{1}{2}(z_b + z_a)} = \frac{\frac{1}{2}(R^2 - z_a^2)}{R z_a + \frac{1}{2}(R^2 + z_a^2)} = \frac{R - z_a}{R + z_a} \cdot \frac{R - \cancel{z_a}}{\cancel{R + z_a}} = \frac{R - z_a}{R + z_a}$$

Ex: $z_a = Ls$, $z_b = \frac{1}{Cs}$, $z_a z_b = R^2 = L/C$, $z_{in} = R$,

$$\frac{V_2}{V_1} = \frac{R - Ls}{R + Ls}$$ which is all-pass: $\left|\frac{V_2}{V_1}(j\omega)\right| = 1$

$\mathcal{L}_{2n}$ functions



Let $\langle v, i \rangle_t = \int_{-\infty}^{\infty} v^T(t)\, i(t)\, dt$

if passive and $\langle v, i \rangle_t$ exists then $\langle v, i \rangle_t \geq 0$ as it is the energy into N over all time

Let

$\langle e, e \rangle_t$ be finite; this is the definition of $\mathcal{L}_{2n}$
$\quad \shortparallel$ i.e. of square integrable vectors
$\|e\|^2$ (strictly needs the type of integral to be specified as Lebesgue [to give the $\mathcal{L}$])

$\|\cdot\|$ is the $\mathcal{L}_{2n}$ norm & $\mathcal{L}_{2n}$ is a "normed" space.

as $e = v + i = 2v^i$; $\langle e, e \rangle = \langle v+i, v+i \rangle = \langle v, v \rangle + \langle i, i \rangle + 2\langle v, i \rangle$

if $e \in \mathcal{L}_{2n}$ and N is passive then every term on the right side is finite $\Rightarrow$ $v \in \mathcal{L}_{2n}$, $i \in \mathcal{L}_{2n}$ and $2v^r \in \mathcal{L}_{2n}$. Thus N defines an $\mathcal{L}_{2n}$ map of incident voltages ($e = 2v^i \in \mathcal{L}_{2n}$) into reflected voltages, $2v^r \in \mathcal{L}_{2n}$. Note N may not allow all $e \in \mathcal{L}_{2n}$, see for example the passive nullator, but in most cases it does. If N is linear and time invariant (and the map is continuous [not so for nullator] as is "almost" always the case); then there exists a scattering (matrix) map (which is a convolution (kernel) operator, $s(t)$)

$$v^{r}(t) = s(t) * v^{n}(t) = \int_{-\infty}^{\infty} s(t-\tau)\, v^{n}(\tau)\, d\tau$$

In actual fact $s(t)$ is a matrix of "distributions" (i.e. things which could be impulses, as would be the case when N is the 2-port gyrator). $s(t)$ will have a Fourier (and Laplace) transform [in the theory of distributions]

Distributions: We consider $\langle v, i\rangle_t$ as above but treat for "functions" f and test functions ϕ as

$$\langle f, \phi\rangle_t = \int_{-\infty}^{\infty} f(t)\phi(t)\,dt \quad \text{when } \int_{-\infty}^{\infty}\cdot \text{ exists}$$

(here use 1×1 matrices)

and generalize when $\int_{-\infty}^{\infty}$ doesn't exist using properties it, $\langle f, \phi\rangle$, would have when it does exist; it is linear in f and ϕ and if ϕ is infinitely differentiable and of "compact support" [like ] then a sequence of ϕ's will be such that

$$\lim_{k\to\infty}\langle f, \phi_k\rangle = \langle f, \lim_{k\to\infty}\phi_k\rangle$$

We then have $\langle f, \cdot\rangle$ as a function of test functions, that is it is a linear continuous functional.

The key idea is that we think of $\langle \mathfrak{f}, \cdot \rangle$ as if it were $\mathfrak{f}$. Note that we take voltmeter readings, that is evaluate $\langle v, \cdot \rangle$ on "voltmeters" φ and can consider all of the readings $\langle v, \varphi \rangle$ to be v.

These linear continuous functionals of test functions φ, $\varphi \in \mathcal{D}$, are the distributions $\mathfrak{f} \in \mathcal{D}'$ where the space of all distributions $\mathcal{D}'$ is the dual space of the test functions $\mathcal{D}$ (just as v and i are duals under $\mathcal{E} = \langle v, i \rangle$).

Examples: 1) all "normal" functions, that is those which allow a true integral $\int \mathfrak{f} \varphi \, dt$ to be calculated, such as e^{-t}, $1(t) =$ unit step

2) The impulse, defined by $\langle \delta, \varphi \rangle = \varphi(0)$

3) The derivative of δ, $\langle \delta', \varphi \rangle = \langle \delta, -\varphi' \rangle = -\varphi'(0)$

4) Not $\frac{1}{t}$, but the "finite part" of $\frac{1}{t}$

5) In this theory $\delta(t) = \frac{d\,1(t)}{dt}$; $\delta' = \frac{d^2 1(t)}{dt^2}$