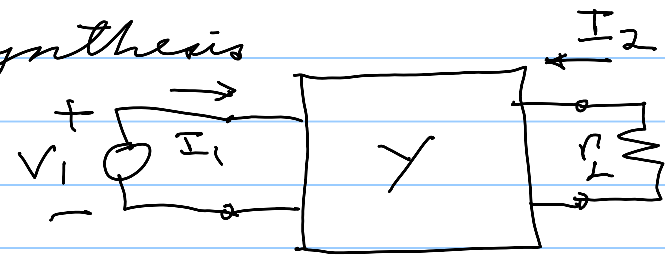
$\frac{V_2}{V_1}$ synthesis



$$+ \quad V_2 = -z_L I_2 \quad -$$

$$\begin{bmatrix} I_1 \\ -y_L V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_2 = y_{21} V_1 + y_{22} V_2 = -y_L V_2 \Rightarrow V_2 = \frac{-y_{21}}{y_{22} + y_L} V_1$$

$$\therefore \frac{V_2}{V_1} = \frac{-y_{21}}{y_{22} + y_L} \Rightarrow \left.\frac{V_2}{V_1}\right|_{y_L = 1} = \frac{-y_{21}}{1 + y_{22}} = \frac{N(s)}{D(s)}$$

Ex: $\dfrac{V_2}{V_1} = \dfrac{c\, s^2}{s^3 + 3s^2 + 3s + 1}$ 2 zeros @ 0, 1 zero @ ∞

$$= \frac{c\, s^2}{(s^3 + 3s) + 3s^2 + 1}$$

$$= \frac{c\left(\dfrac{s^2}{s^3+3s}\right)}{1 + \dfrac{3s^2+1}{s^3+3s}} = \frac{-y_{21}}{1 + y_{22}}$$

choose $y_{22} = \dfrac{3s^2+1}{s^3+3s} = \dfrac{3s^2+1}{s(s^2+3)} = \dfrac{3(s^2+1/3)}{s(s^2+3)}$

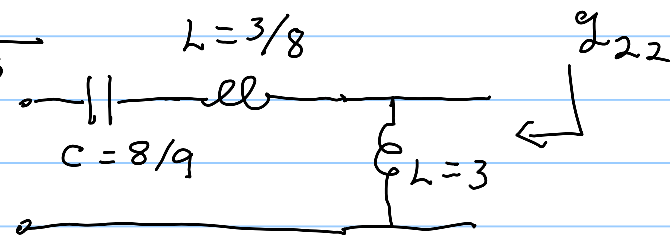
EE 610
10/27/03
P.3

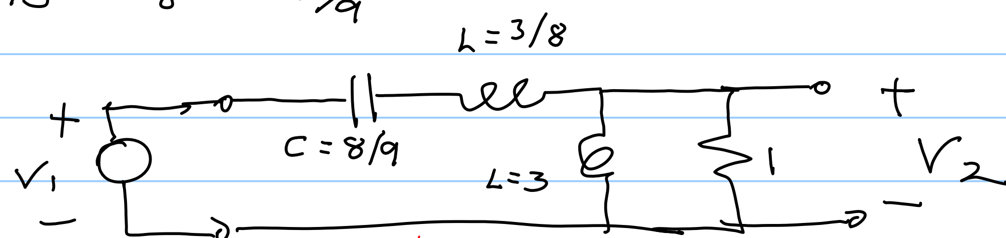
Synthesize y_{22} to give 0's of y_{21}:

$$y_{22} = \frac{1/3}{s} + \frac{ks}{s^2+3} \qquad k = \frac{3(s^2+1/3)}{s^2}\Bigg|_{s^2=-3} = \frac{-8}{-3} = 8/3$$

$$= \frac{1/3}{s} + \frac{8s/3}{s^2+3}$$

$$y \rightarrow \frac{1}{\dfrac{s^2+3}{8s/3}} = \frac{1}{\dfrac{3}{8}s + \dfrac{1}{8s/9}} \quad z \rightarrow$$





0@0   0@∞   0@0 = zeros of transmission

EE 610
10/27/03
p.4

$$\frac{V_2}{V_1} = \frac{\dfrac{1}{1+\frac{1}{3}s}}{\dfrac{1}{1+\frac{1}{3s}} + \dfrac{1}{\frac{8}{9}s} + \frac{3}{8}s} = \frac{\dfrac{3s}{1+3s}}{\dfrac{3s}{1+3s} + \dfrac{9+3s^2}{8s}}$$

$$= \frac{24s^2}{24s^2 + (1+3s)(9+3s^2)} = \frac{24s^2}{24s^2 + 9 + 27s + 9s^3 + 3s^2}$$

$$= \frac{24s^2/9}{s^3+3s^2+3s+1} \Rightarrow k = \frac{8}{3}$$

Hurwitz polynomial: $P(s)$

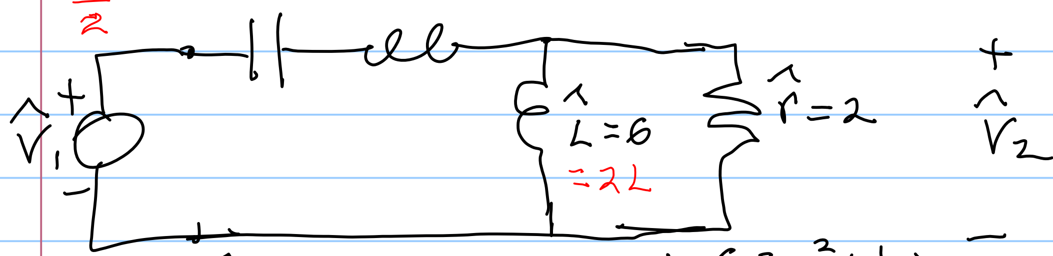
strictly Hurwitz = no zeros of $P(s)$ in $\sigma \geq 0$

non strictly Hurwitz = no zeros of $P(s)$ in $\sigma > 0$

Denormalization for $r_{load} = 2$

EE 610
10/27/03
p. 5

$\frac{C}{2} = \hat{C} = 4/9 \quad \hat{L} = 3/4 = 2L$

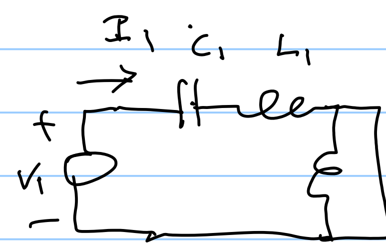


$$\hat{y}_{22} = \frac{1}{2} y_{22} = \frac{1}{2}\left(\frac{3s^2+1}{s^3+3s}\right)$$

$$-\hat{y}_{21} = \frac{c}{2}\cdot\frac{s^2}{s^3+3s} = \frac{4}{3}\,\frac{s^2}{s^3+3s}$$

$$y_{11} = \frac{I_1}{V_1}\Bigg|_{V_2=0} = \frac{1}{L_1 s + \frac{1}{C_1 s}} = \frac{C_1 s}{L_1 C_1 s^2 + 1}$$

$$= \frac{8/9\, s}{\frac{3}{9} s^2 + 1} = \frac{8s}{3s^2+9}$$



EE 610
10/27/03
p. 6

$$Y = \begin{bmatrix} \dfrac{8s}{3s^2+9} & \dfrac{-8s/3}{3s+s^3} \\ \dfrac{-8s/3}{3s+s^3} & \dfrac{3s^2+1}{s(s^2+3)} \end{bmatrix} = \frac{1}{s}\begin{bmatrix} 0 & 0 \\ 0 & 1/3 \end{bmatrix}$$

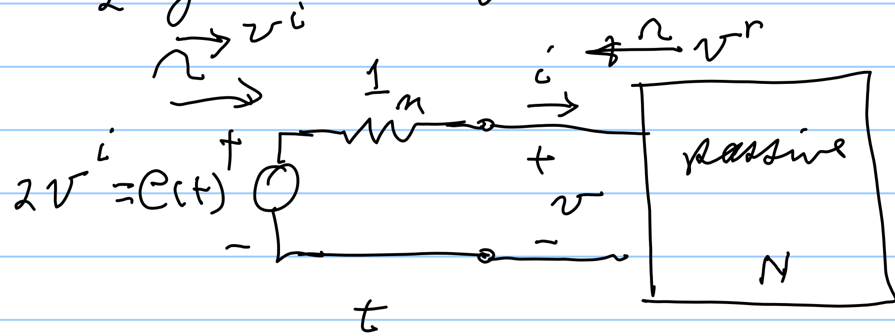
$$+ \frac{s}{s^2+3}\begin{bmatrix} 8/3 & -8/3 \\ -8/3 & 8/3 \end{bmatrix}$$

$$= \frac{1}{s}\begin{bmatrix} 0 & 0 \\ 0 & 1/3 \end{bmatrix} + \frac{1}{s+j\sqrt{3}}\begin{bmatrix} 4/3 & -4/3 \\ -4/3 & 4/3 \end{bmatrix} + \frac{1}{s-j\sqrt{3}}\begin{bmatrix} 4/3 & -4/3 \\ -4/3 & 4/3 \end{bmatrix}$$

$\delta[Y]$ = degree of $Y(s) = 1+1+1 = 3$

(McMillan degree = sum of ranks of residue matrices if simple poles)

$\mathcal{L}_2$ functions, $v^i(t)$ & $v^r(t)$



$$\mathcal{E}(t) = \int_{-\infty}^{t} v(\tau)\, i(\tau)\, d\tau$$

if ≥ 0 is passive for all v & i's allowed

|| energy into N

EE 610
10/27/03
p. 8

Consider v^i as in $\mathcal{L}_2$ that is

$$\int_{-\infty}^{\infty} v^{iT}(\tau)\, v^i(\tau)\, d\tau \quad \text{exists (\& is} \geq 0)$$

$2v^r = v - i$, $2v^i = v + i$

→ set $\|v^i\|^2 = \int_{-\infty}^{\infty} v^{iT} v^i \, d\tau$

$$\|v^i\|^2 = \left\| \frac{v+i}{2} \right\|^2 = \frac{\|v\|^2 + \|i\|^2}{2^2} + 2\int_{-\infty}^{\infty} v^T i \, d\tau$$

⇒ if passive $\|v\|^2$ is finite as is $\|i\|^2$