EE610_102203

Note Title 10/22/2003

S matrix P. 1

$$v^r = S v^i$$

$2v^i = v + i$ $\quad$ $A(\omega) v = B(\omega) i$

$2v^r = v - i$ $\quad$ $\Downarrow$

$\Downarrow$ $\quad$ $A(v^i + v^r) = B(v^i - v^r)$

$v = v^i + v^r$ $\Rightarrow$ $\quad$ $\Downarrow$

$i = v^i - v^r$ $\quad$ $(B - A) v^i = (B + A) v^r$

$$\Rightarrow \quad S = (B+A)^{-1}(B-A)$$

$$Z = A^{-1}B, \quad Y = B^{-1}A$$

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$$S = (AA^{-1}[B+A])^{-1}(AA^{-1}[B-A])$$

$$= (A[A^{-1}B+1_n])^{-1}(A[A^{-1}B-1_n])$$

$$= (Z+1_n)^{-1}A^{-1}A[Z-1_n]$$

or $S = (Z+1_n)^{-1}(Z-1_n)$ this is also

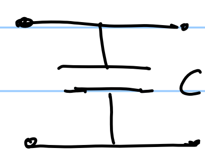
$$= (Z-1_n)(Z+1_n)^{-1}$$

assume true that $(Z-1_n)(Z+1_n)^{-1} = (Z+1_n)^{-1}(Z-1_n)$

$$(Z+1_n)(Z-1_n) = (Z-1_n)(Z+1_n)$$

$$Z^2+Z-Z-1_n = (Z^2-Z+Z-1_n)$$

ex:



$$Z_c = \frac{1}{sC}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$$

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$$S = (Z + 1_2)^{-1}(Z - 1_2) \; ; \quad Z + 1_2 = \frac{1}{sC}\begin{bmatrix} sC+1 & 1 \\ 1 & sC+1 \end{bmatrix}$$

$$\Delta_{1_Z} = \frac{1}{(sC)^2}\left[(1+sC)^2 - 1 \right] = \frac{sC}{(sC)^2}(2+sC) = \frac{(2+sC)}{sC}$$

$$S = \frac{\cancel{sC}}{2+sC}\begin{bmatrix} \frac{1+sC}{sC} & \frac{-1}{sC} \\ \frac{-1}{sC} & \frac{1+sC}{sC} \end{bmatrix}\begin{bmatrix} 1-sC & 1 \\ 1 & 1-sC \end{bmatrix}\frac{1}{\cancel{sC}}$$

$$= \frac{1}{2+sC}\begin{bmatrix} 1-(sC)^2+sC-sC-1 & 1+sC-1+sC \\ -1+sC+1+sC & -1+1+sC-sC-(sC)^2 \end{bmatrix}\frac{1}{sC}$$

$$= \frac{\cancel{sC}}{2+sC}\begin{bmatrix} -sC & 2 \\ 2 & -sC \end{bmatrix} \times \frac{1}{\cancel{sC}} = \frac{1}{2+sC}\begin{bmatrix} -sC & 2 \\ 2 & -sC \end{bmatrix}$$

lossless: $S(s)S^T(-s) = 1_n$ here $n = 2$

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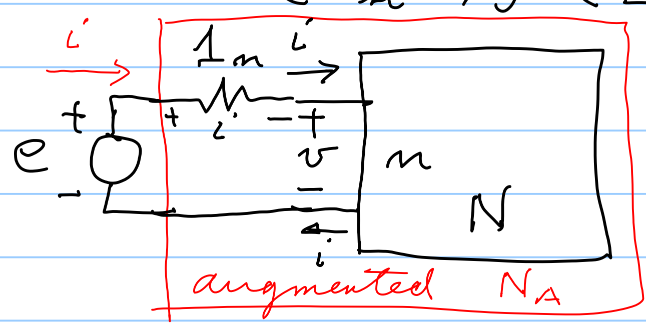
$$S^T(-s) = \frac{\cancel{-sc}}{2-sc}\begin{bmatrix} sc & 2 \\ 2 & sc \end{bmatrix}$$

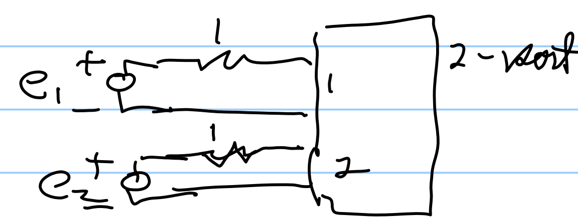
$$S S_*^T = \frac{sc}{2+sc}\cdot\frac{-sc}{2-sc}\begin{bmatrix} -sc & 2 \\ 2 & -sc \end{bmatrix}\begin{bmatrix} sc & 2 \\ 2 & sc \end{bmatrix}.$$

$$= \frac{\cancel{-(sc)^2}}{(2+sc)(2-sc)}\begin{bmatrix} -(sc)^2+4 & 0 \\ 0 & 4-(sc)^2 \end{bmatrix} = -\cancel{(sc)^2}\, 1_2$$

also $S = (1_n + Y)^{-1}(1_n - Y)$



$$2v^i = v + i = e$$
$$2v^r = v - i$$



assume $i = Y_a \cdot e \Rightarrow v = e - i = (1_n - Y_a)e$

$i = Y_a e$
$v = (1_n - Y_a)e$ $\Rightarrow A v = B i \Rightarrow CAv = CBi$ if C^{-1} exists

$\equiv \quad (1_n - Y_a) i = (1_n - Y_a) Y_a e = Y_a (1_n - Y_a) e = Y_a v$

$A = Y_a, \quad B = 1_n - Y_a$

$S = (B+A)^{-1}(B-A) = 1_n^{-1}(1_n - 2Y_a)$

$S = (1_n - 2Y_a)$

Lossless: $2P_{ave}(\omega) = V^{T*} I + I^{*T} V$

$V = V^i + V^r$
$I = V^i - V^r$

$= (V^i + V^r)^{T*}(V^i - V^r)$
$+ (V^i - V^r)^{T*}(V^i + V^r)$
$= V^{iT*}V^i - \cancel{V^{iT*}V^r} + \cancel{V^{rT*}V^i} - V^{rT*}V^r$
$+ V^{iT*}V^i - \cancel{V^{rT*}V^i} + \cancel{V^{iT*}V^r} - V^{rT*}V^r$

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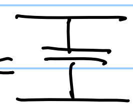
$$2\,P_{ave}(\omega) = 2\,V^{iT*}V^{i} - 2\,V^{rT*}V^{r}$$

$$P_{ave} = V^{i*T}V^{i} - V^{i*T}S^{T*}S\,V^{i} = \|V^{i}\|^{2} - \|V^{r}\|^{2}$$

$$= V^{i*T}\left[1_n - S^{T*}(j\omega)S(j\omega)\right]V^{i}$$

for all V^i complex ≥ 0 if passive $= 0$ if lossless

$\Rightarrow$ if passive no poles on $j\omega$ axis

(our ex c  $\Rightarrow$ poles at $s = -2/c$ in $S(s) = \frac{1}{2+sc}\begin{bmatrix} -sc & 2 \\ 2 & -sc \end{bmatrix}$

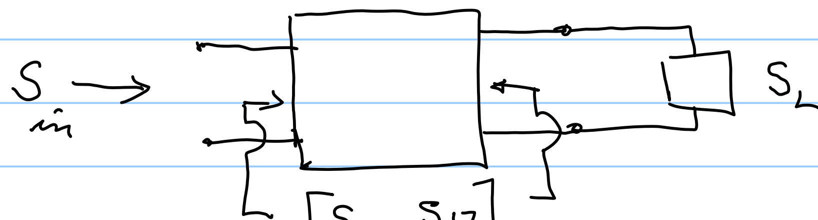
if passive $S(s)$ is bounded real

a) $S(\sigma)$ is real for $\sigma > 0$

b) $S(s)$ is analytic in $\sigma > 0$

c) $1_n - S^{T*}(s)\,S(s)$ is positive semi-definite

Termination of N in a load



$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$