EE610_102003

Note Title 10/16/2003

Semistate Equations

$$E\dot{x} = Ax + Bu$$

$$y = Cx$$

Ex

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = [\qquad\qquad] x$$

$$\left\{ \begin{bmatrix} 0 & s \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\} X(s) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} U(s)$$

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$$\begin{bmatrix} -1 & s \\ 0 & -1 \end{bmatrix}^{-1} = \frac{1}{1}\begin{bmatrix} -1 & -s \\ 0 & -1 \end{bmatrix}$$

$$X(s) = \begin{bmatrix} -1 & -s \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} U(s) = \begin{bmatrix} -1-s \\ -1 \end{bmatrix} u$$

$$Y(s) = \begin{bmatrix} -1 & +1 \end{bmatrix} X(s) = s\, U(s)$$

$$= \begin{bmatrix} -1 & +1 \end{bmatrix}\begin{bmatrix} -(1+s) \\ -1 \end{bmatrix} U(s) = [s]\, U(s)$$

$\therefore$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\dot{x} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} -1 & 1 \end{bmatrix} x \implies \text{differentiator}$$

$$\frac{Y(s)}{U(s)} = s$$

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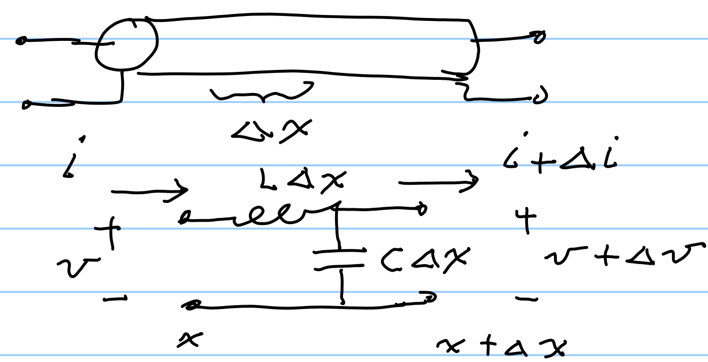
More general case (nonlinear & time-varying) $\quad E\dot{x} = a(x,t) + Bu$

$$y = Cx$$

Toward the scattering matrix

look at a lossless transmission lines



$$v - (v + \Delta v) = L\,\Delta x \frac{di}{dt} \Rightarrow -\frac{\Delta v}{\Delta x} = L \frac{di}{dt}$$

$$i - (i + \Delta i) = C\,\Delta x \frac{d(v+\Delta v)}{dt} \Rightarrow -\frac{\Delta i}{\Delta x} = C \frac{dv}{dt} + \underbrace{\frac{d\Delta v}{dt}}_{\text{2nd order}}$$

$\Downarrow$ 0

$$\lim_{\Delta x \to 0}: \quad \frac{dv}{dx} = -L \frac{di}{dt}, \quad \frac{di}{dx} = -C \frac{dv}{dt}$$

$$\left.\begin{aligned} v(t,x) &= v^i\left(t - \frac{x}{\nu}\right) + v^r\left(t + \frac{x}{\nu}\right) \\ z_0\, i(t,x) &= v^i(t - x/\nu) - v^r(t + x/\nu) \end{aligned}\right\} \text{assume then}$$

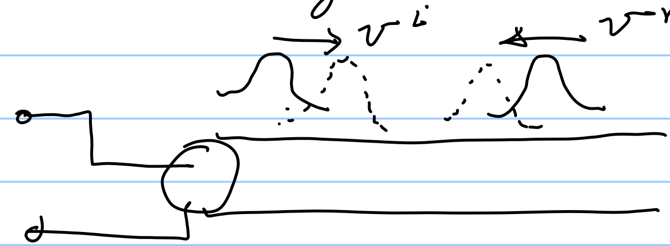
$$\frac{dv}{dx} = -\frac{1}{\nu} \dot{v}^i(t - x/\nu) + \frac{1}{\nu} \dot{v}^r(t + x/\nu) =$$

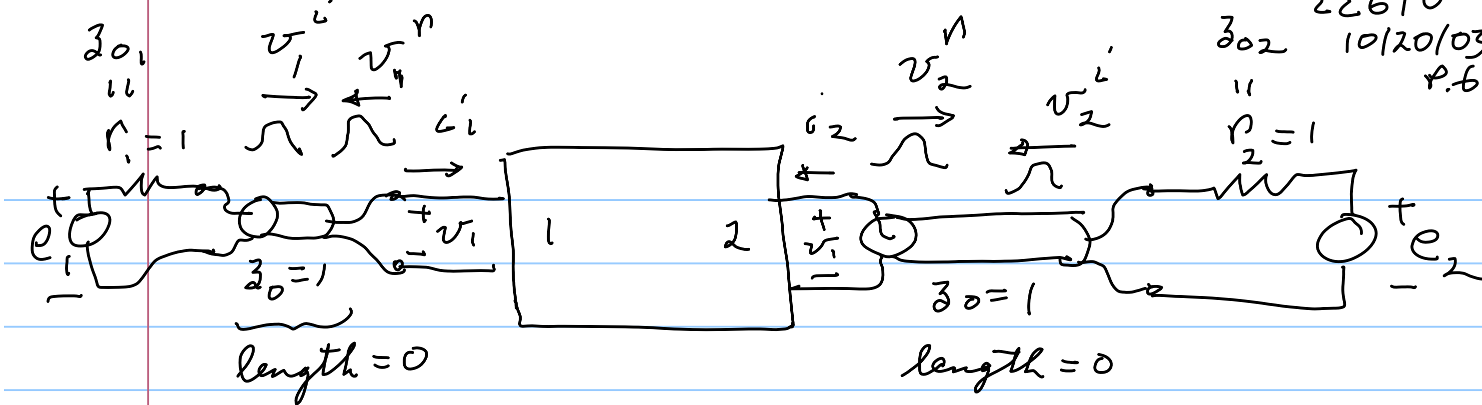
$$z_0 \frac{di}{dt} = \dot{v}^i(t - x/\nu) - \dot{v}^r(t + x/\nu) = -\nu\, dv/dx$$

$$\frac{dv}{dx} = -\frac{Z_0}{\nu}\frac{di}{dt} \quad ; \quad Z_0 = \sqrt{\Delta L \cdot \frac{1}{\Delta C}} \; ; \; \nu = \frac{1}{\sqrt{LC}}$$

$$\frac{Z_0}{\nu} = \sqrt{\frac{L}{C}} \cdot \sqrt{LC} = L$$

∴ the solution assumed is valid (after check di/dx eq.)





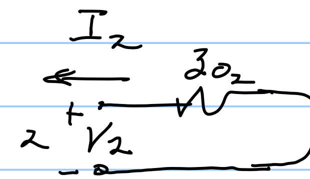
$$v_1 = v_1^i + v_1^r \quad ; \quad v_2 = v_2^i + v_2^r$$

$$z_0 i_1 = i_1 = v_1^i - v_1^r \qquad i_2 = v_2^i - v_2^r$$

$$\left.\begin{array}{l} v = v^i + v^r \\ \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} i = Z_0 i = v^i - v^r \end{array}\right\} \begin{array}{l} 2v^i = v + Z_0 i \\ 2v^r = v - Z_0 i \end{array}$$

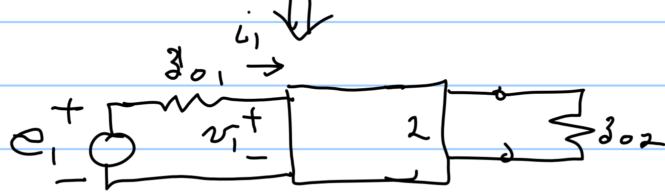
$S(s)$ = scattering matrix $\quad V^r = S V^i$

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \Rightarrow S_{21} = \left.\frac{V_2^r}{V_1^i}\right|_{V_2^i = 0} =$$



$$2V_2^i = V_2 + z_o I_2 = 0 \Rightarrow V_2 = -z_{o_2} I_2$$

$$= E_2 = 0$$



$$2v_1^i = v_1 + z_{o_1} i_1 = e_1$$

$$2v_2^r = v_2 - z_{o_2} i_2$$

$$= 2v_2$$

$$S_{21} = \frac{V_2}{E_1/2} = 2\frac{V_2}{E_1}$$ $\Rightarrow$ a voltage gain of a terminated 2-port

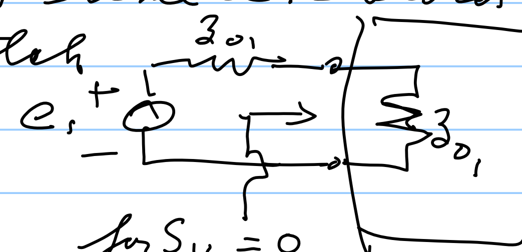
what does $S_{11} = 0$ mean? nothing sent in comes back at you.

$$S_{11} = \left.\frac{V_1^r}{V_1^i}\right|_{V_2^i = 0} = \qquad V_2^i = 0 \Rightarrow E_2 = 0$$

$$2V_1^r = V_1 - Z_0 I_1$$

$$\therefore \text{ if } V_1^i = E_1/2 \neq 0 \Rightarrow V_1^r = 0 \Rightarrow V_1 = Z_{01} I_1$$

that is the input looks like a resistor of value Z_{01} i.e. a match



for $S_{11} = 0$

Power relations:

$$P_{ave}(\omega) = Re\left(V^{T*} I\right) = \frac{V^{T*} I + I^{T*} V}{2}$$

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$$V = V^i + V^r$$

$$I = V^i - V^r \quad \text{if } Z_0 = \text{identity}$$

$$2P_{ave}(\omega) = \left(V^{iT*} + V^{rT*}\right)\left(V^i - V^r\right) + \left(V^{i*T} - V^{rT*}\right)\left(V^i + V^r\right)$$

$$= 2V^{iT*}V^i - 2V^{rT*}V^r$$

$$P_{ave} = V^{iT*}V^i - V^{iT*}S^{T*}SV^i = V^{iT*}\left[1_n - S^{T*}S\right]V^i$$

$S \rightarrow$ Bounded real if passive.