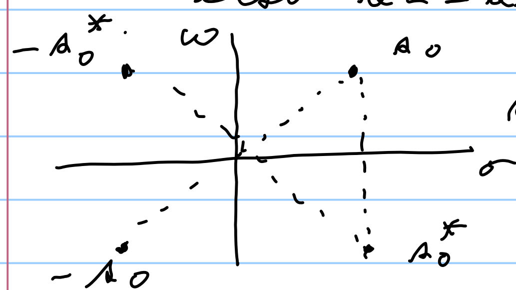
Complex zeros of $Ev\, z(s)$:

assume $z(s)$ real coefficients, rational positive-real.

$$Ev\, z(s) = \frac{1}{2}\left(z(s) + z(-s)\right) = \frac{1}{2}\left(z + z_{*}\right)$$

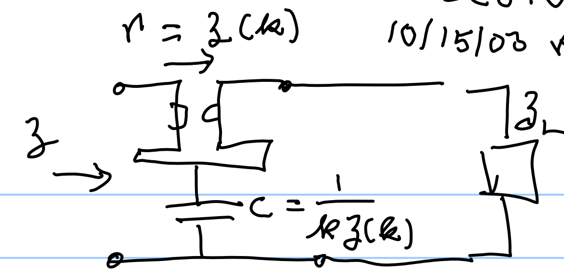
if $= 0$ at $s = s_0$, s_0^* is also a zero

also $s = -s_0$ & $-s_0^*$.

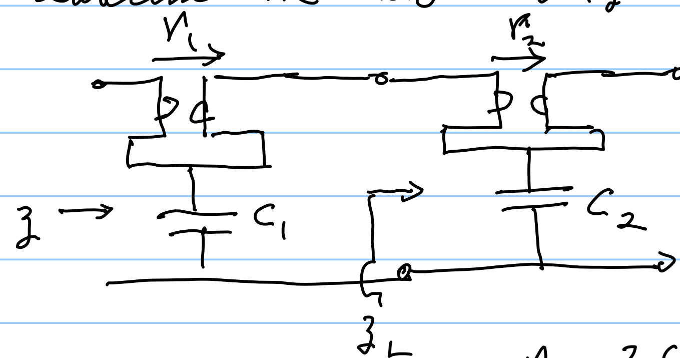


s-plane; $s = \sigma + j\omega$

$$z_L(s) = z(k)\left[\frac{k\,z(s) - s\,z(k)}{k\,z(k) - s\,z(s)}\right]$$



assume $k = s_0 = \sigma_0 + j\omega_0$, $\sigma_0 > 0$



$$\longrightarrow z_{LL} = z_L(k^*)\left[\frac{k^* z_L(s) - s\,z_L(k^*)}{k^* z_L(k^*) - s\,z_L(s)}\right]$$

$r_1 = z(k)$

$C_1 = \frac{1}{k\,z(k)}$

$r_2 = z_L(k^*)$

$C_2 = \frac{1}{k^* z_L(k^*)}$

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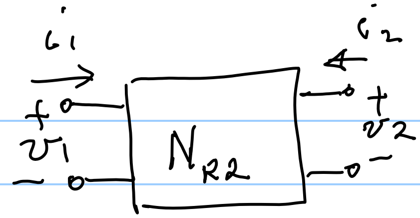
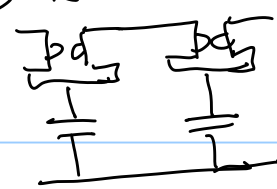
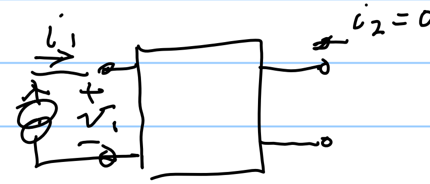
@ k k*



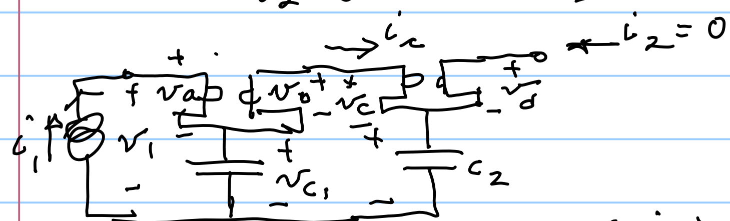


find Z for N_{R2}

$$z_{11} = \frac{v_1}{i_1}\Big|_{i_2=0}$$



$v_1 = z_{11}$ if $i_1 = 1$



$$\begin{bmatrix} v_c \\ v_d \end{bmatrix} = \begin{bmatrix} 0 & r_2 \\ -r_2 & 0 \end{bmatrix} \begin{bmatrix} i_c \\ i_d = 0 \end{bmatrix}$$

$\Rightarrow v_c = 0$

$$v_1 = v_a + v_{c_1} = r_1(-i_c) + \frac{1}{sC_1}[i_1 - i_c] = \frac{i_1}{sC_1} - \left(r_1 + \frac{1}{sC_1}\right) i_c$$

$$v_{c_2} = \frac{1}{sC_2}(i_c) = v_b + v_{c_1} = -r_1 i_1 + \frac{1}{sC_1}[i_1 - i_c]$$

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or $\left(\frac{1}{sC_2} + \frac{1}{sC_1}\right) i_c = \left(-r_1 + \frac{1}{sC_1}\right) i_1$

$$\Rightarrow i_c = \frac{-r_1 + \frac{1}{sC_1}}{\frac{1}{sC_1} + \frac{1}{sC_2}} \Rightarrow v_1 = \frac{i_1}{sC_1} - \frac{\left(r_1 + \frac{1}{sC_1}\right)\left(-r_1 + \frac{1}{sC_1}\right)}{\frac{1}{sC_1} + \frac{1}{sC_2}} i_1$$

gives $z_{11} = \frac{1}{s(C_1+C_2)} + \frac{s r_1^2 C_1 C_2}{C_1 + C_2}$

$$z_{22} = \frac{1}{s(C_1+C_2)} + \frac{s r_2^2 C_1 C_2}{C_1 + C_2}$$

$$z_{12} = \frac{\left(r_1 + \frac{1}{sC_1}\right)\left(r_2 + \frac{1}{sC_2}\right)}{\frac{1}{s}\left(1/C_1 + 1/C_2\right)} ; \quad z_{21} = \frac{\left(-r_1 + 1/sC_1\right)\left(-r_2 + \frac{1}{sC_2}\right)}{\frac{1}{s}\left(1/C_1 + 1/C_2\right)}$$

$$\frac{1}{C_1 + C_2} = \frac{1}{\frac{1}{kz(k)} + \frac{1}{k^* z_L(k^*)}} = \frac{\operatorname{Im}(k z(k^*))}{\operatorname{Im}(k^2)}$$ which is real

also

$$z_{12} = \frac{s[r_1 r_2 C_1 C_2] + [r_1 C_1 + r_2 C_2] + 1/s}{C_1 + C_2}$$

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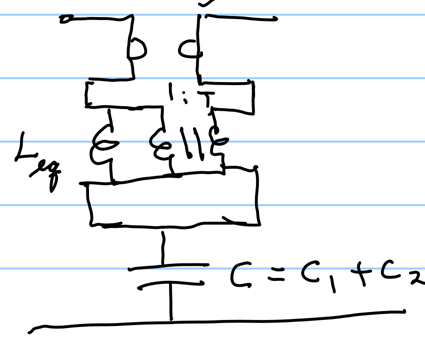
$$z_{21} = \frac{s[r_1 r_2 C_1 C_2] - [r_1 C_1 + r_2 C_2] + 1/s}{C_1 + C_2}$$

$$Z(s)\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \frac{1}{C_1 + C_2}\left\{ s\, C_1 C_2 \begin{bmatrix} r_1^2 & r_1 r_2 \\ r_1 r_2 & r_2^2 \end{bmatrix} + (r_1 C_1 + r_2 C_2)\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + \frac{1}{s}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right\}$$

$$r = (r_1 C_1 + r_2 C_2)/(C_1 + C_2) = 2\,\mathrm{Re}(1/k)\,\frac{\mathrm{Im}(k\, z(k^*))}{\mathrm{Im}(k^2)}$$



$$\frac{\mathrm{Im}(k\, z(k))}{\mathrm{Im}(k^2)} = L_{eq} = r_1^2 C_1 C_2/(C_1 + C_2)$$

$$T^2 L_{eq} = \frac{r_2^2 C_1 C_2}{C_1 + C_2} \Rightarrow T^2 = \left(\frac{r_2}{r_1}\right)^2$$

T = turns ratio $= r_2/r_1$