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Note Title 10/13/2003

oscillators

$$\ddot{x} + \omega_0^2 x = 0 \qquad x(0),\ \dot{x}(0)$$

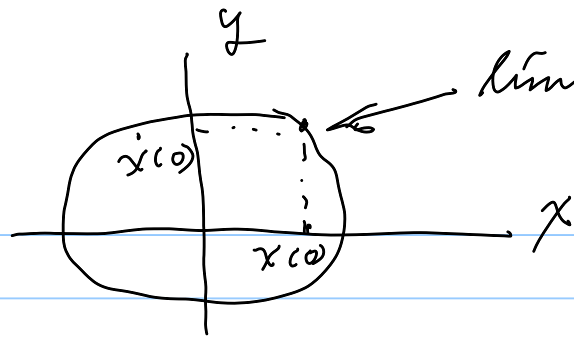
$$\dot{y} = -\omega_0^2 x = \ddot{x}$$

$$\int^t \dot{y}(\tau)\,d\tau = y(t) = \int^t \frac{d\dot{x}(\tau)}{d\tau}\,d\tau = \dot{x}(t)$$

$$\dot{x} = y$$
$$\dot{y} = -\omega_0^2 x$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \quad \underline{x} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= A\underline{x}$$



limit cycles vary with initial conditions

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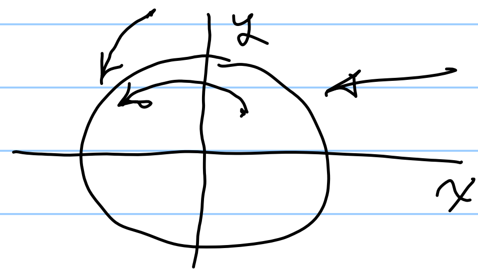
Van der Pol oscillator

$$\ddot{x} + \epsilon(x^2 - 1)\dot{x} + \omega_0^2 x = 0 \qquad \epsilon > 0$$



limit cycle – independent of initial conditions

structurally stable

set up for Spice

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$$\dot{y} = -\omega_0^2 x = \ddot{x} + \epsilon(x^2 - 1)\dot{x}$$

$$\int^t \dot{y} = y = \dot{x} + \int^t \epsilon(x^2)\dot{x}\,d\tau - \int^t \epsilon\,\dot{x}\,d\tau$$

$$= \dot{x} + \epsilon\frac{x^3}{3} - x\epsilon$$

note $\int^t \epsilon\,\dot{x}\,d\tau = \int^t \epsilon \frac{dx(\tau)}{d\cancel{\tau}}\,d\cancel{\tau} = \epsilon \int^t dx = \epsilon x$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} y - \epsilon(x)\left(\frac{x^2}{3} - 1\right) \\ -\omega_0^2 x \end{bmatrix}$$