ee610_100803

Note Title 10/8/2003

semistate:

$$E\frac{dx}{dt} = Ax + Bu; \quad y = Cx$$

x = semistate; E could be singular;
assume for now E, A, B, C are constant matrices

will choose for now $x = \begin{bmatrix} v_t \\ i_\ell \end{bmatrix}$

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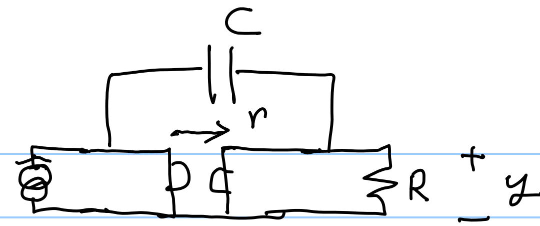
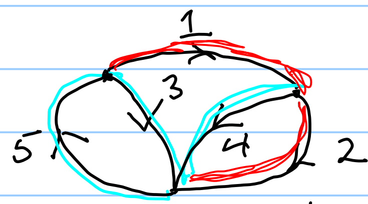
Example

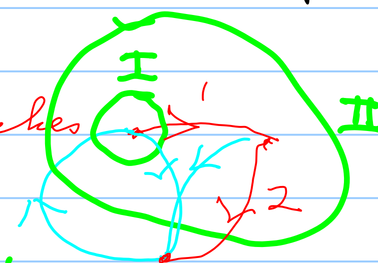
$u = i_s$





— = tree branches

— = link branches



KCL

$\mathrm{I} \Rightarrow 0 = i_{1b} + i_{3b} - i_{5b}$

$\mathrm{II} \Rightarrow 0 = i_{2b} + i_{3b} + i_{4b} - i_{5b}$

$$\Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \left[\begin{array}{cc|ccc} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 1 & 1 & -1 \end{array}\right] \begin{bmatrix} i_1 \\ i_2 \\ \hline i_3 \\ i_4 \\ i_5 \end{bmatrix}_b$$

KVL

$3 \rightarrow \mathrm{I}$

$4 \rightarrow \mathrm{II}$

$5 \rightarrow \mathrm{III}$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \left[\begin{array}{cc|ccc} -1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} v_1 \\ v_2 \\ \hline v_3 \\ v_4 \\ v_5 \end{bmatrix}_b$$

assume an admittance matrix for all branches

$$Y_{bb} = \begin{bmatrix} Cs & 0 & 0 & 0 & 0 \\ 0 & G & 0 & 0 & 0 \\ 0 & 0 & 0 & -g & 0 \\ 0 & 0 & g & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Y_{gys} = Z_{gys}^{-1} = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix}, \quad g = \frac{r}{r^2} = \frac{1}{r}$$

$$Z_{gys} = \begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix}$$

$$Av = Bi \Rightarrow Y_{bb} v = 1_b i \qquad b = 5$$

$$v = v_b - e \Rightarrow v = v_b = \mathcal{C}^T v_t \Rightarrow A\mathcal{C}^T v_t = B(i_b - j)$$

$$i = i_b - j \qquad\qquad = 1_5(\mathcal{J}^T i_\ell - j)$$

$$\text{or} \quad \left[A\mathcal{C}^T \,\middle|\, -B\mathcal{J}^T \right] \begin{bmatrix} v_t \\ i_\ell \end{bmatrix} = Ae - Bj$$

$$Y_{bb}\, e^T = A e^T = \begin{bmatrix} Cs & & & & \\ & G & & & \\ & & 0 & -g & \\ & & g & 0 & \\ & & & & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} Cs & 0 \\ 0 & G \\ 0 & -g \\ g & g \\ 0 & 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|ccc} Cs & 0 & 1 & 0 & -1 \\ 0 & G & 1 & 1 & -1 \\ 0 & -g & -1 & 0 & 0 \\ g & g & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right] \begin{bmatrix} v_1 \\ v_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}_b = \frac{0}{s} - \frac{1}{s} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ i_1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} C & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{dx}{dt} = \begin{bmatrix} 0 & 0 & -1 & 0 & 1 \\ 0 & -G & -1 & -1 & 1 \\ 0 & g & +1 & 0 & 0 \\ -g & -g & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} u$$

$$y = [\,0 \;\; 1 \;\; 0 \;\; 0 \;\; 0\,]\, x$$

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use