EE610_100603

Note Title 10/6/2003

sensitivity using the adjoint

$S_\alpha^{T(\alpha)}$ = sensitivity of $T(\alpha)$ w.r.t parameter α

$$= \frac{\alpha}{T(\alpha)} \frac{\partial T(\alpha)}{\partial \alpha} = \frac{(\partial T(\alpha))/T(\alpha)}{(\partial \alpha)/\alpha}$$

Ex $T(\alpha) = \frac{V_2}{V_s} = \frac{3\alpha + R}{2R\alpha + 1}$, S_R^T

$$\frac{\partial T}{\partial R} = \frac{1}{2R\alpha+1} - \frac{(3\alpha+R)(2\alpha)}{(2R\alpha+1)^2} = \frac{(\cancel{2R\alpha}+1) - 6\alpha^2 - \cancel{2R\alpha}}{(2R\alpha+1)^2}$$

$$S_R^T = \frac{R}{T}\frac{\partial T}{\partial R} = \frac{R}{\frac{3\alpha+R}{2R\alpha+1}} \cdot \frac{-6\alpha^2+1}{(2R\alpha+1)^2} = \frac{R(-6\alpha^2+1)}{(3\alpha+R)(2R\alpha+1)}$$

Using the adjoint:

Def of adjoint of a network N = original
N^a = adjoint

assume N & N^a are finite with
the same graph — from graph theory $e^a J^T = 0$ $t \times l$

$$i_b^{aT} v_b - i_b^T v_b^a = 0$$

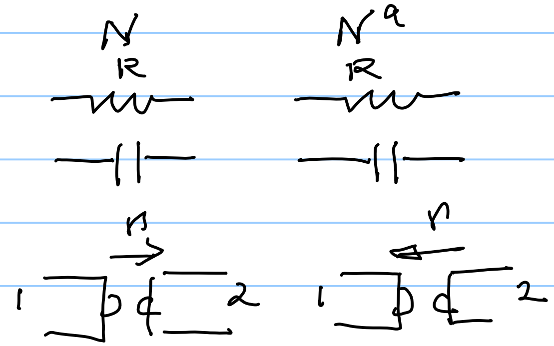
assume $i_b = Y_{b\times b} v_b$ & $i_b^a = Y_{b\times b}^a v_b^a$

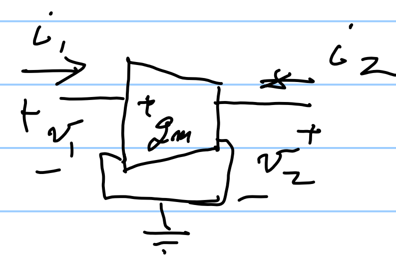
$$v_b^{aT} Y_{b\times b}^{aT} v_b - v_b^T Y_{b\times b}^T v_b^a = 0$$

$$v_b^{aT} Y_{b\times b}^{aT} v_b - v_b^{aT} Y_{b\times b} v_b = 0$$

$$\Rightarrow v_b^{aT}\left[Y_{b\times b}^{aT} - Y_{b\times b}\right] v_b = 0$$ for all v_b's & v_b^a's

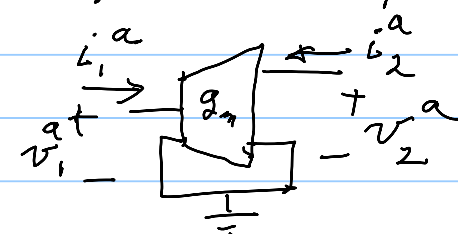
$$\Rightarrow Y^a_{b\times b} = Y^T_{b\times b}$$





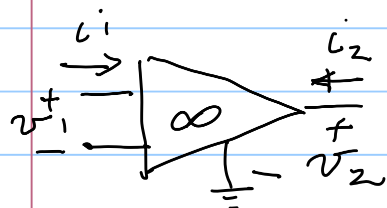
$$Y = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix}$$

$$Y^a \Rightarrow Y^T = Y^a = \begin{bmatrix} 0 & g_m \\ 0 & 0 \end{bmatrix}$$



ideal op-amp:

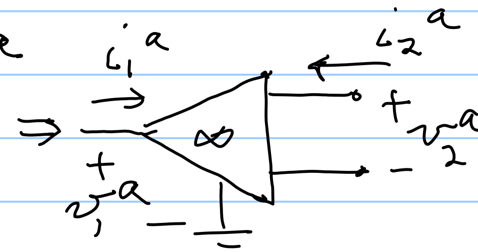
$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$



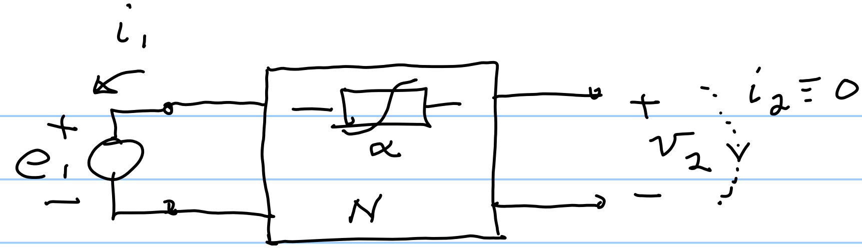
$$\underset{\overset{\shortparallel}{0}}{i_1^a}\, v_1 + i_2^a v_2 - \underset{\overset{\shortparallel}{0}}{i_1}\, v_1^a - i_2 v_2^a = 0$$

as v_2 & i_2 can be anything then this is satisfied by $i_2^a = 0 = v_2^a$

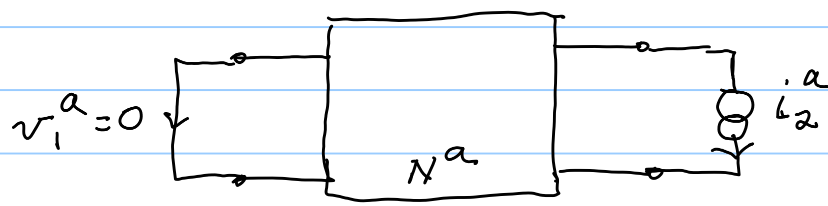
$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}^a = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}^a \Rightarrow$$



For sensitivity



$$T(\alpha) = \frac{v_2}{e_1}$$



$$e_1 i_1^a + v_2 i_2^a - \underbrace{v_1^a}_{\substack{0 \\ \text{by choice} \\ \text{of } v_1^a}} i_1 - v_2^a \underbrace{i_2}_{\substack{0 \\ \text{(original} \\ v_2 \text{ measured as open circuit)}}} = -\left[i_N^{aT} v_N - i_N^T v_N^a \right]$$

$$\underbrace{\frac{\partial}{\partial\alpha}}_{=0}\left(e_1 \dot{i}_1^{a} + v_2 \dot{i}_2^{a} - \underset{0''}{v_1^a} \dot{i}_1 - \underset{0''}{v_2^a} \dot{i}_2\right) = -\frac{\partial}{\partial\alpha}\left[\dot{i}_N^{aT} v_N - \dot{i}_N^{T} v_N^{a}\right]$$

as e_1 = fixed in α

$\frac{\partial \dot{i}_1^a}{\partial\alpha} = 0$ as nothing in N^a is to vary

$$= \frac{\partial v_2}{\partial\alpha} \dot{i}_2^a + v_2 \underbrace{\frac{\partial \dot{i}_2^a}{\partial\alpha}}_{0}$$

as nothing in N^a varies

$$\frac{\partial v_2}{\partial\alpha} = \left.\frac{\partial T(\alpha)}{\partial\alpha}\right|_{e_1=1} = \frac{-1}{\dot{i}_2^a}\left[\dot{i}_N^{aT}\frac{\partial v_N}{\partial\alpha} - \frac{\partial \dot{i}_N^T}{\partial\alpha} v_N^a\right]$$

now $\frac{\partial \dot{i}_N}{\partial\alpha} = \frac{\partial}{\partial\alpha}\left[Y_N \cdot v_N\right]$ assuming N described by Y_N

$$= \frac{\partial Y_N}{\partial\alpha} v_N + Y_N \frac{\partial v_N}{\partial\alpha}$$

$$i_N^{aT} \frac{\partial v_N}{\partial \alpha} - \frac{\partial i_N^T}{\partial \alpha} v_N^a = v_N^{aT} Y_N^{aT} \frac{\partial v_N}{\partial \alpha} - \left[v_N^T \frac{\partial Y_N^T}{\partial \alpha} + \frac{\partial v_N^T}{\partial \alpha} Y_N^T \right] v_N^a$$

$$= - v_N^T \frac{\partial Y_N^T}{\partial \alpha} v_N^a + \left[v_N^{aT} \underbrace{\left\{ Y_N^{aT} - Y_N \right\}}_{0} \frac{\partial v_N}{\partial \alpha} \right]$$

0 by definition of the adjoint.

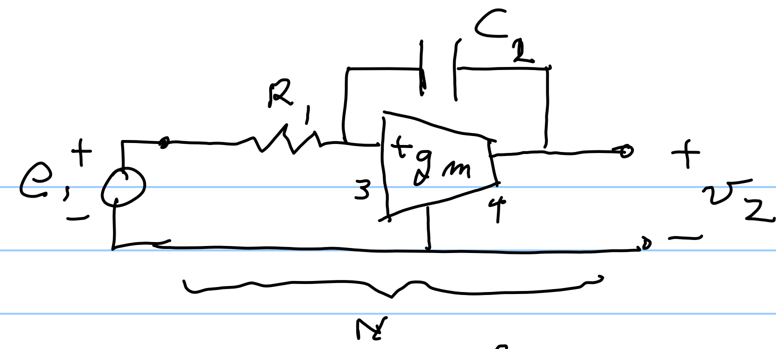
$\Rightarrow$

$$\frac{\partial v_2}{\partial \alpha} = \frac{-1}{i_2^a} \left\{ - v_N^{aT} \frac{\partial Y_N}{\partial \alpha} \cdot v_N \right\} = \frac{1}{i_2^a} v_N^{aT} \frac{\partial Y_N}{\partial \alpha} \cdot v_N$$

voltage

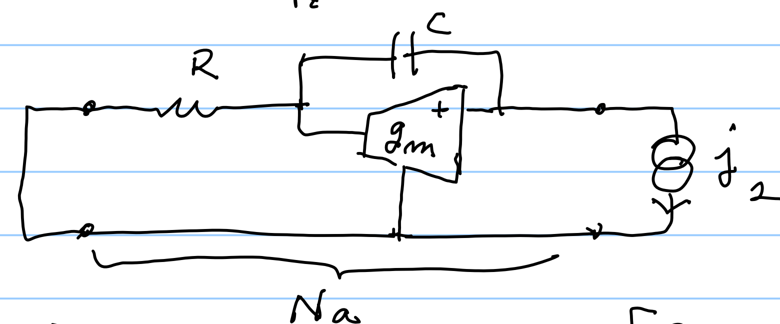
if choose $e_1 = 1$, $i_2^a = 1$ we solve for the terms that multiply the $\frac{\partial Y_N}{\partial \alpha}$ entries in N & $\hat{N}^a$

Example



$$T(s) = \frac{v_2}{e_1}$$

$$S^{T}_{g_m}$$



$$\frac{\partial Y_N}{\partial g_m}\,; \quad Y_N = \begin{bmatrix} G_1 & & & \\ & sC_1 & & \\ & & 0 & 0 \\ & & g_m & 0 \end{bmatrix} \Rightarrow \frac{\partial Y_N}{\partial g_m} = \begin{bmatrix} 0 & & & \\ & 0 & & \\ & & 0 & 0 \\ & & 1 & 0 \end{bmatrix}$$

EE610
10/06/03
p. 9

derive v_3 in N & v_4^a in N^a