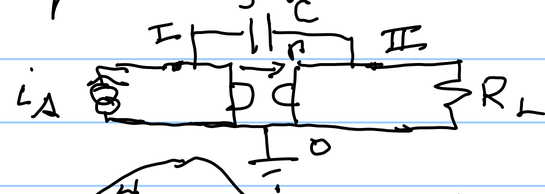
EE610_092903

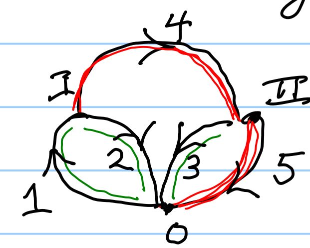
Note Title 9/29/2003

Graph theory for circuit analysis

chapter 3, p. 91

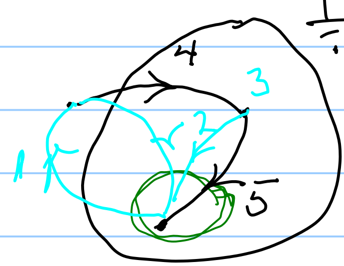


⇒



= tree

= cotree
= limbs
= links

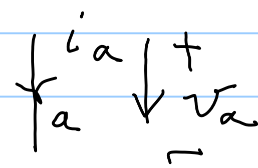


sphere 1

cut set 1 = branches 1, 2, 4

cut set 2 = branches 1, 2, 3, 5

generic branch



$$\Rightarrow i_b = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}_b , \quad v_b = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}_b$$

KCL: sphere 1: $0 = -i_1 + i_2 \quad + i_4$

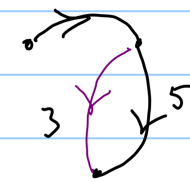
$\quad$ 2: $0 = -i_1 + i_2 + i_3 \quad + i_5 \Rightarrow 0 = \mathcal{C}_t \, i_b$

t = # of tree branches = 2, n = # of nodes = 3

b = # of branches = 5, $\mathcal{C}$ = cut set matrix = $t \times b$

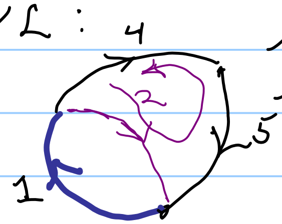
$$\mathcal{C} = \left[\; \cdots \;\middle|\; 1_t \right] \Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \left[\begin{array}{ccc|cc} \underbrace{\begin{matrix} -1 & 1 & 0 \\ -1 & 1 & 1 \end{matrix}}_{K_{t \times l}} & & & \underbrace{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}}_{1_2} & \end{array}\right] \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}_b$$

Cut set equations:



KVL:

loop 1	$0 =$	v_1		$+ v_4$	$+ v_5$
loop 2	$0 =$		v_2	$- v_4$	$- v_5$
loop 3	$0 =$		v_3		$- v_5$



l = # of links = 3 $\quad 0_l = \mathcal{T} \cdot v_b$; $\mathcal{T}$ = tie set matrix $l \times b$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 \end{array}\right] \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}_b \quad ; \quad \mathcal{T} = \left[\, 1_l \;\vdots\; \dots \,\right]$$

$$\mathcal{C} = \left[\, K \;\vdots\; 1_t \,\right], \quad \mathcal{T} = \left[\, 1_l \;\vdots\; -K^T \,\right]$$

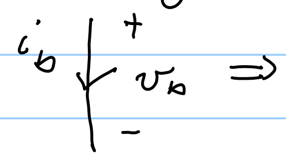
$$\Rightarrow \quad \mathcal{C}\,\mathcal{T}^T = \underset{t \times l}{\left[\, K \;\vdots\; 1_t \,\right]} \begin{bmatrix} 1_l \\ \hdashline -K \end{bmatrix} = \left[\, K - K \,\right] = 0_{t \times l}$$

$$0_l = v_l - K^T v_t \implies v_b = \begin{bmatrix} v_l \\ v_t \end{bmatrix} = \begin{bmatrix} K^T \\ 1_t \end{bmatrix} v_t = \mathcal{C}^T v_t$$

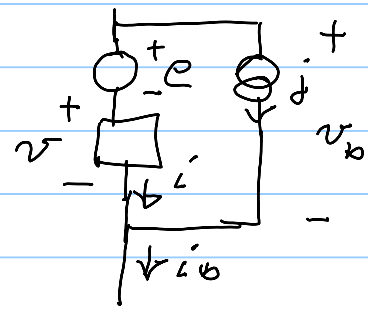
Laws of connection are KVL & KCL

$v_b = \mathcal{C}^T v_t \,, \quad i_b = \mathcal{D}^T i_l$

Laws of elements:



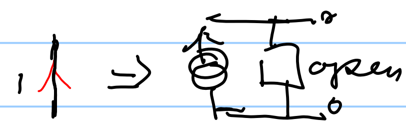
$\Rightarrow$



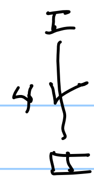
$v_b = e + v$

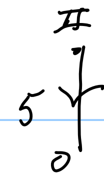
$i_b = j + i$

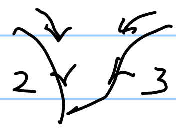
i versus v laws: assume every branch is part of an admittance matrix



admittance of v_1 vs i_1 $\Rightarrow$ $i_1 = 0 \cdot v_1$



$= $  $\quad y = Cs$,  $y = \frac{1}{R_L}$



$$Y = \begin{bmatrix} 0 & -\frac{1}{r} \\ \frac{1}{r} & 0 \end{bmatrix}$$

all other entries zero

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 0 & & & & \\ & 0 & -\frac{1}{r} & & \\ & \frac{1}{r} & 0 & & \\ & & & Cs & \\ & & & & \frac{1}{R_L} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} \Rightarrow i = Y_{b\times b}\, v$$

$e = 0$ ← 5-vector

$$j = \begin{bmatrix} i_s \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$ ← corrected

$$v_b = v$$

$$i_b = j + i = j + Y_{b\times b}\, v$$

$$= j + Y_{bb} v_b = j + Y_{bb} e^T v_t$$

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also $0_t = e\, i_b \Rightarrow$ $\underbrace{e\, i_b}_{=0} = e\, j + e\, Y_{b\times b}\, e^T v_t$

$$-e\, j = e\, Y_{b\times b}\, e^T \cdot v_t \Rightarrow$$ t equations in t unknowns

added material

$$-e\, j = -\begin{bmatrix} -1 & 1 & 0 & 1 & 0 \\ -1 & 1 & 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} i_s \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} +i_s \\ +i_s \end{bmatrix}$$

$$e\, Y_{b\times b}\, e^T = \begin{bmatrix} -1 & 1 & 0 & 1 & 0 \\ -1 & 1 & 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & & & & \\ & 0 & -g & & \\ & g & 0 & & \\ & & & sC & \\ & & & & G_L \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \; ; \; g = 1/r, \quad G_L = 1/R_L$$

added
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$$= \begin{bmatrix} 0 & 0 & -g & sC & 0 \\ 0 & g & -g & 0 & G_L \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} sC & -g \\ g & G_L \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} i_s \\ i_s \end{bmatrix} = \begin{bmatrix} sC & -g \\ g & G_L \end{bmatrix} \begin{bmatrix} v_4 \\ v_5 \end{bmatrix} \quad ;$$

if desire input z need $v_{in} = -v_1$

by $v_b = C^T v_t$

$$v_{in} = -v_1 = -\begin{bmatrix} -1 & -1 \end{bmatrix} \begin{bmatrix} v_4 \\ v_5 \end{bmatrix}$$

$$= v_4 + v_5$$

$$= \frac{sC + G_L}{sCG_L + g^2} i_s = z\, i_s$$

solving

$$\begin{bmatrix} v_4 \\ v_5 \end{bmatrix} = \frac{1}{sCG_L + g^2} \begin{bmatrix} G_L & g \\ -g & sC \end{bmatrix} \begin{bmatrix} i_s \\ i_s \end{bmatrix}$$

$$= \frac{i_s}{sCG_L + g^2} \begin{bmatrix} G_L + g \\ sC - g \end{bmatrix}$$