ee610_092403

Note Title 9/24/2003

LC - 1st Foster P.1

$$Z(s) = -Z(-s) = \frac{k_0}{s} + k_\infty s + \sum_{i=1}^{n} \frac{k_i s}{s^2 + \omega_i^2}, \quad k_i \geq 0$$

let $s = j\omega$; $Z(j\omega) = jX(\omega)$

$$X(\omega) = -\frac{k_0}{\omega} + k_\infty \omega + \sum_{i=1}^{n} \frac{k_i \omega}{\omega_i^2 - \omega^2}$$

$$\frac{dX(\omega)}{d\omega} = \frac{k_0}{\omega^2} + k_\infty + \sum_{i=1}^{n} \left[\frac{k_i}{\omega_i^2 - \omega^2} + \frac{2k_i \omega \omega}{(\omega_i^2 - \omega^2)^2} \right]$$

$$= \frac{k_0}{\omega^2} + k_\infty + \sum_{i=1}^{n} \left[\frac{k_i(\omega_i^2 + \omega^2)}{(\omega_i^2 - \omega^2)^2} \right] \geq 0$$

(continuous except at poles

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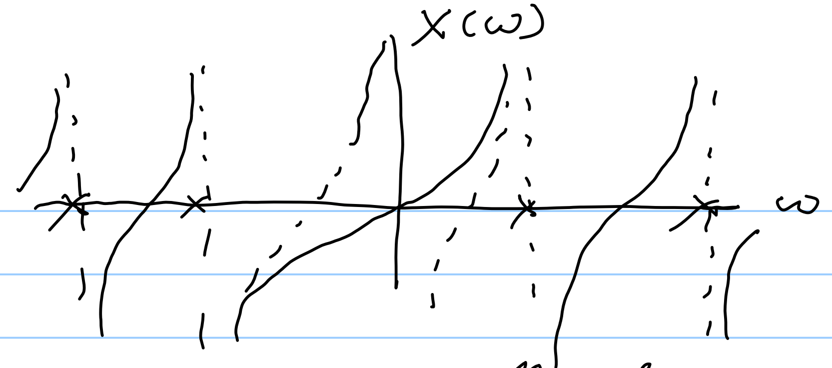
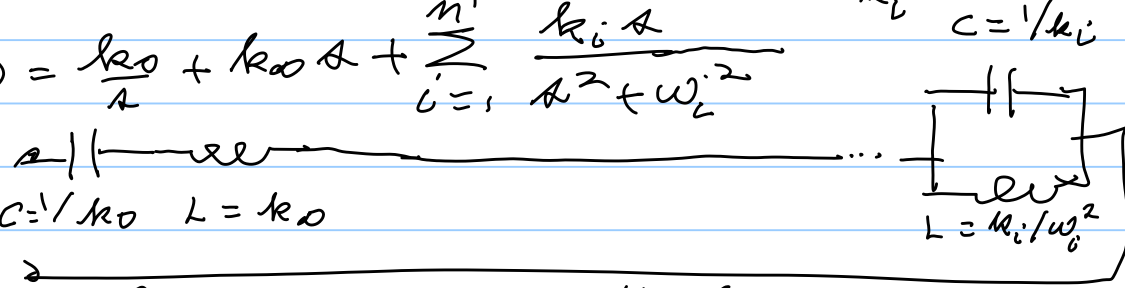


between poles there is a zero

$$\frac{1}{\frac{s^2+\omega_i^2}{k_i s}}$$

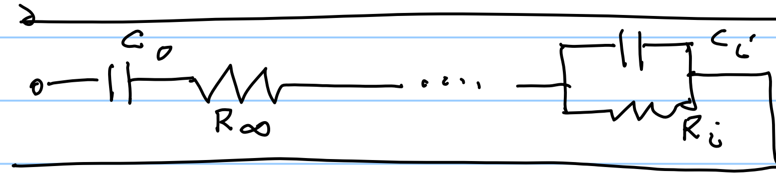
$$Z(s) = \frac{k_0}{s} + k_\infty s + \sum_{i=1}^{n} \frac{k_i s}{s^2+\omega_i^2}$$

Z_{LC}



$\Rightarrow$

Z_{RC}



$$Z_{RC} = \frac{k_0}{s} + k_\infty + \sum_{i=1}^{n} \frac{1}{C_i s + \frac{1}{R_i}} = \frac{k_0}{s} + k_\infty + \sum_{i=1}^{n} \frac{k_i}{s + \sigma_i}$$

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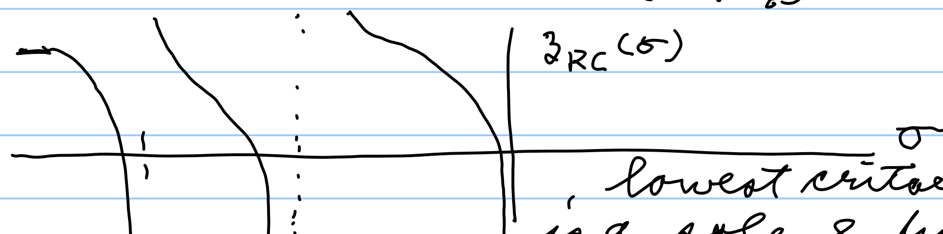
this is a partial fraction expansion for Z_{RC}

$Z_{RC}(s)\big|_{s=\infty} = k_\infty$ or no pole at ∞, can be a pole at 0

$$\frac{d\,Z_{RC}(\sigma)}{d\sigma} = -\frac{k_0}{\sigma^2} + \sum_{i=1}^{n} \frac{-k_i}{(\sigma+\sigma_i)^2} < 0$$

see poles and zeros alternate



lowest critical frequency is a pole & highest a zero

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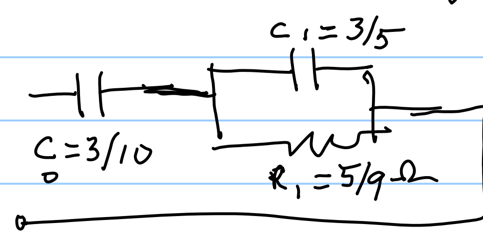
Ex: $z(s) = \dfrac{5(s+2)}{s(s+3)}$ this is a PR RC $z(s)$

1st Foster $= \dfrac{10/3}{s} + \dfrac{5/3}{s+3} \Rightarrow$

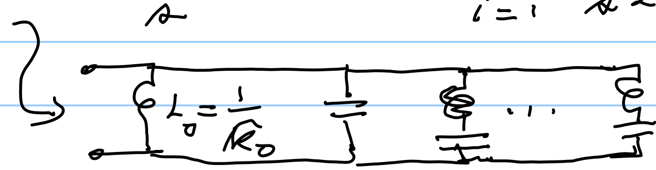


2nd Foster

$$y(s) = \frac{1}{z(s)} = \frac{s(s+3)}{5(s+2)} = \frac{1}{5}s + \frac{-\frac{2}{5}(-2+3)}{s+2} = \frac{1}{5}s + \frac{-2/5}{s+2}$$

trouble as a negative residue

$$y_{LC}(s) = \frac{\hat{k}_0}{s} + \hat{k}_\infty s + \sum_{i=1}^{\hat{n}} \frac{\hat{k}_i s}{s^2 + \hat{\omega}_i^2}$$



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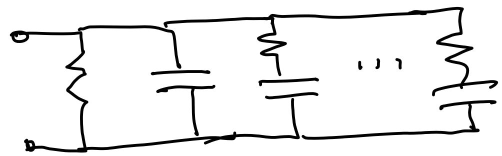
$Y_{RC} \rightarrow$



$$\frac{c_i s}{r_i c_i s + 1} = \frac{\frac{1}{r_i} s}{s + 1/r_i c_i}$$

$$Y_{RC} = \hat{k}_0 + \hat{k}_\infty s + \sum \frac{1}{r_i + \frac{1}{c_i s}}$$

$$= \hat{k}_0 + \hat{k}_\infty s + \sum_{i=1}^{\hat{n}} \frac{\hat{k}_i s}{s + \hat{\sigma}_i}$$

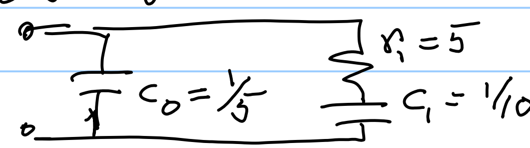
as not a partial fraction expansion

$$\frac{Y_{RC}}{s} = \frac{\hat{k}_0}{s} + \hat{k}_\infty + \sum_{i=1}^{\hat{n}} \frac{\hat{k}_i}{s + \hat{\sigma}_i}$$

2nd Fosters

$$Y_{RC} = \frac{s(s+3)}{5(s+2)} \Longrightarrow \frac{Y_{RC}}{s} = \frac{(s+3)}{5(s+2)} = \frac{1}{5} + \frac{1/5}{s+2}$$

$$\therefore Y_{RC}(s) = \frac{s}{5} + \frac{\frac{1}{5} s}{s+2}$$



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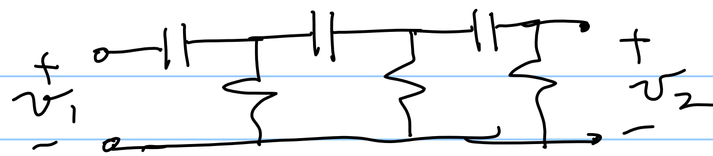
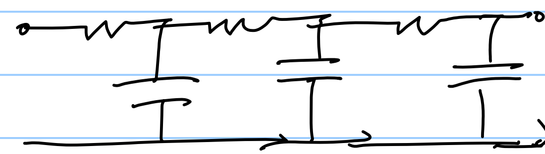
1st Cauer high pass



zeros of transmission at $s = 0$

2nd Cauer low pass



zeros of transmission at $s = \infty$