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Note Title 9/22/2003

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State variable equations – chapter 5

$$\dot{x} = Ax + Bu \qquad x = \text{state (vector)}$$
$$y = Cx + Du \qquad A, B, C, D \text{ constant matrices}$$

u = input, y = output

$x \rightarrow$ inductor currents, capacitor voltages

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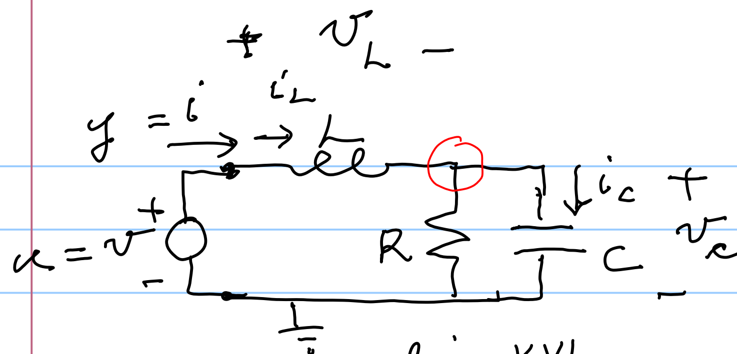

$$v_L = L\frac{di_L}{dt} \overset{KVL}{=} v - v_c \Rightarrow \frac{di_L}{dt} = -\frac{1}{L}v_c + \frac{1}{L}v$$

$$i_c = C\frac{dv_c}{dt} \overset{KCL}{=} i_L + \left(-\frac{1}{R}v_c\right) \Rightarrow \frac{dv_c}{dt} = -\frac{1}{RC}v_c + \frac{1}{C}i_L$$

$$x = \begin{bmatrix} i_L \\ v_c \end{bmatrix}; \quad \frac{dx}{dt} = \begin{bmatrix} di_L/dt \\ dv_c/dt \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}\begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}\begin{bmatrix} v \end{bmatrix}$$

$$y = [i] = [i_L] = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} i_L \\ v_c \end{bmatrix}$$

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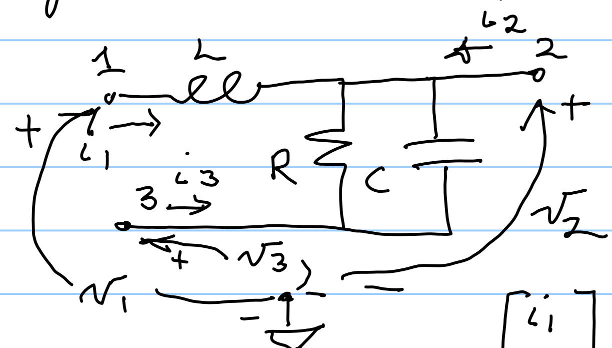
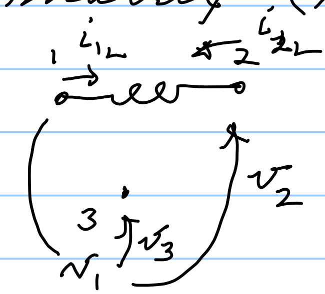
$y = i_L = i, \quad u = v$

Indefinite admittance matrix (s domain)





$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}_L = \begin{bmatrix} \frac{1}{sL} & -\frac{1}{sL} & 0 \\ -\frac{1}{sL} & \frac{1}{sL} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

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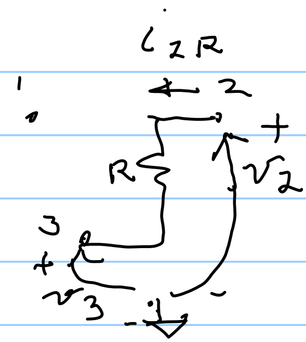
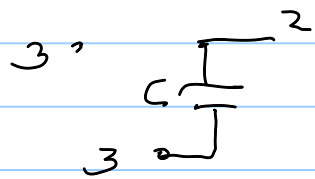


$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}_R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{R} & -\frac{1}{R} \\ 0 & -\frac{1}{R} & \frac{1}{R} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$



$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}_C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & sC & -sC \\ 0 & -sC & sC \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}_L + \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}_R + \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}_C = \overbrace{\begin{bmatrix} \frac{1}{sL} & -\frac{1}{sL} & 0 \\ -\frac{1}{sL} & \frac{1}{sL} + \frac{1}{R} + sC & -\frac{1}{R} - sC \\ 0 & -\frac{1}{R} - sC & \frac{1}{R} + sC \end{bmatrix}}^{\text{indefinite admittance}} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

move ground to circuit $\Rightarrow v_3 = 0$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{sL} & -\frac{1}{sL} \\ -\frac{1}{sL} & \frac{1}{Ls} + \frac{1}{R} + sC \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

ignore i_3

for input admittance set $i_2 = 0$ and eliminate v_2 (= internal node elimination

$$\frac{1}{sL} v_1 = \left[\frac{1}{Ls} + \frac{1}{R} + sC\right] v_2 \Rightarrow v_2 = \frac{1}{sL} \cdot \frac{1}{\frac{1}{Ls} + \frac{1}{R} + sC} \cdot v_1$$

$$i_1 = \left[\frac{1}{sL} - \frac{1}{sL} \cdot \frac{1}{sL} \cdot \frac{1}{\frac{1}{Ls} + \frac{1}{R} + sC}\right] v_1 = y_{in}(s) \cdot v_1$$

$$= \left[\frac{\cancel{\frac{1}{sL} \cdot \frac{1}{Ls}} + \frac{1}{sL} \cdot \frac{1}{R} + \frac{sC}{sL} - \cancel{\frac{1}{sL} \cdot \frac{1}{sL}}}{\frac{1}{Ls} + 1/R + sC}\right] v_1 = \frac{\frac{1}{R} + sC}{s^2 LC + \frac{L}{R} s + 1} v_1$$

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$$Y_{in}(s) = \frac{C}{LC}\left(\frac{s + {}^1/_{RC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}\right)$$

here

$$s^2 + \frac{\omega_0}{Q}s + \omega_0^2 = s^2 + 2\zeta\omega_n s + \omega_n^2 \; ; \quad \omega_0 = \omega_n = \frac{1}{\sqrt{LC}}$$

$$\frac{\omega_0}{Q} = \frac{1}{RC} \Rightarrow Q = \omega_0 RC = R\sqrt{\frac{C}{L}}$$

poles are at $s_{1,2} = -\frac{\omega_0}{2Q} \pm \frac{1}{2}\sqrt{\left(\frac{\omega_0}{Q}\right)^2 - 4\omega_0^2} = \frac{\omega_0}{2Q}\left[-1 \pm \sqrt{1-4Q^2}\right]$