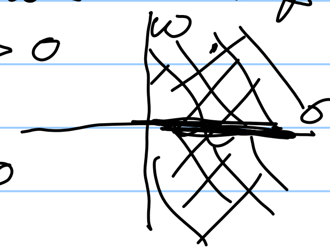
EE 610 09/17/03

Note Title 9/17/2003

P. 338 → positive real functions

By definition $f(s)$ is positive real if



1) $f(s)$ is analytic in $\sigma > 0$
   (stability)

2) $f(s)$ is real for $s = \sigma$, $\sigma > 0$
   (uses real elements)

3) $\mathrm{Re}\, f(s) \geq 0$ in $\sigma > 0$
   (passivity)

for finite circuits $f(s)$ is rational

1) → all poles in $\sigma \leq 0$; 2) → real coefficients

3) $\mathrm{Re}\, f(s) = \frac{1}{2}\left(f(s) + f^*(s)\right) = \frac{1}{2}\left(f(s) + f(s^*)\right)$

Ex: Is $f(s) = \frac{2s-3}{s+2}$ positive real?

no as $\frac{1}{f(s)}$ has a pole at $s = 3/2$ in $\sigma > 0$

and if f is positive real then so is $1/f(s)$
check the definition

1) pole only at $s = -2 \Rightarrow$ analytic in $\sigma > 0$

2) $f(\sigma) = \frac{2\sigma - 3}{\sigma + 2}$ is real for real $\sigma > 0$

3) $2 \mathrm{Re}\, f(s) = \frac{2(\sigma + j\omega) - 3}{\sigma + j\omega + 2} + \frac{2(\sigma - j\omega) - 3}{(\sigma - j\omega) + 2}$

$$= \frac{[2(\sigma + j\omega) - 3][\sigma - j\omega + 2] + [2(\sigma - j\omega) - 3][\sigma + j\omega + 2]}{\sigma^2 + \omega^2 + 4\sigma + 4}$$

EE 610
09/17/03
p.3

$\Rightarrow 2(\sigma^2+\omega^2) - 3\sigma + 4\sigma - 6 \sim -6$ for small $\sigma > 0$, $\omega = 0$

$\therefore$ Re $f(s)$ can be < 0 in $\sigma > 0$

so this $f(s) = \frac{2s-3}{s+2}$ is not positive real

if rational and positive real $\rightarrow$ PR

also PR means physically realizable

$\Rightarrow$ can synthesize by a finite number of passive elements.

Ex: non-PR but positive real

$z(s) = \frac{1}{\sqrt{s}}$ is such

$z(s) = \tanh(s)$ is also

Definition of passive; an n-port, v, i are variables $\Rightarrow P = \mathrm{Re}\left(V^{*T}(j\omega)\, I(j\omega)\right) \geq 0$ for almost all ω

take $\hat{g}(s) = I^{*T}(j\omega)\, Z^{T*}(j\omega)\, I(j\omega)\Big|_{\omega = s/j}$

$P = \mathrm{Re}\left(I^{*T}(j\omega)\, V(j\omega)\right) \geq 0$ = one definition of passive

$g(s) = I^{*T} Z(s) I$

assume $Z(s)$ analytic in $\sigma > 0$; $\underline{I}$ = constant n-vector

desire to show $\mathrm{Re}\, g(s) \geq 0$ in $\sigma > 0$

use the maximum modulus theorem

$e^{-g(s)} \Rightarrow$ analytic in $\sigma > 0$

$$\left|e^{-g(s)}\right| = \left|e^{-\mathrm{Re}\, g(s) - j\,\mathrm{Im}\, g(s)}\right| = e^{-\mathrm{Re}\, g(s)}$$

$\Rightarrow \mathrm{Re}\, g(j\omega) \leq \mathrm{Re}\, g(s)$ in $\sigma > 0$.

for any I

$$\Rightarrow 2\,\mathrm{Re}\, g(s) = I^{T*}\left[Z(s) + Z^{T*}(s)\right] I \geq 0 \quad \text{in } \sigma > 0$$

$$\frac{Z(s) + Z^{T*}(s)}{2} = \text{Hermitian of } Z(s)$$

is positive semi definite $\Longleftrightarrow$ passive

Note: gyrator: $Z(s) = \begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix}$ $\quad r$ real

$$\mathrm{Herm}\, Z = \frac{1}{2}\left\{ Z + Z^{T} \right\} = \frac{1}{2}\begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Ex: $Z = \begin{bmatrix} \frac{1}{Cs} & \frac{1}{Cs} + r \\ \frac{1}{Cs} - r & \frac{1}{Cs} \end{bmatrix} \Rightarrow$ is positive real & PR if r is real & $C > 0$

EE610
09/17/03
p. 6

$$Z(s) + Z^{T*}(s) = \begin{bmatrix} \frac{1}{Cs} & \frac{1}{Cs} + n \\ \frac{1}{Cs} - n & \frac{1}{Cs} \end{bmatrix} + \begin{bmatrix} \frac{1}{Cs^*} & \frac{1}{Cs^*} - n \\ \frac{1}{Cs^*} + n & \frac{1}{Cs^*} \end{bmatrix} = \underbrace{\left(\frac{1}{Cs} + \frac{1}{Cs^*}\right)}_{\frac{2\sigma}{C(\sigma^2+\omega^2)}} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

is positive semidefinite in $\sigma > 0$.